Funsor Documentation

Release 0.0

Uber AI Labs

Jan 24, 2021

Funsor Core:

CHAPTER 1

Operations

1.1 Operation classes

class CachedOpMeta

Bases: `type`

Metaclass for caching op instance construction.

class Op(*fn*, *, *name=None*)

Bases: `multipledispatch.dispatcher.Dispatcher`

classmethod subclass_register(**pattern*)

make_op(*fn=None*, *parent=None*, *, *name=None*, *module_name='funsor.ops'*)

declare_op_types(*locals_*, *all_*, *name_*)

class UnaryOp(*fn*, *, *name=None*)

Bases: `funsor.ops.op.Op`

class TransformOp(*fn*, *, *name=None*)

Bases: `funsor.ops.op.UnaryOp`

set_inv(*fn*)

Parameters **fn** (*callable*) – A function that inputs an arg `y` and outputs a value `x` such that `y=self(x)`.

set_log_abs_det_jacobian(*fn*)

Parameters **fn** (*callable*) – A function that inputs two args `x, y`, where `y=self(x)`, and returns `log(abs(det(dy/dx)))`.

static inv(*x*)

static log_abs_det_jacobian(*x*, *y*)

1.2 Builtin operations

abs = ops.abs

add = ops.add

and_ = ops.and_

atanh = ops.atanh

eq = ops.eq

exp = ops.exp

ge = ops.ge

getitem = ops.GetitemOp(0)
Op encoding an index into one dimension, e.g. `x[:,:,y]` for offset of 2.

gt = ops.gt

invert = ops.invert

le = ops.le

lgamma = ops.lgamma

log = ops.log

log1p = ops.log1p

lt = ops.lt

matmul = ops.matmul

max = ops.max

min = ops.min

mul = ops.mul

ne = ops.ne

neg = ops.neg

nullop = ops.nullop
Placeholder associative op that unifies with any other op

or_ = ops.or_

pow = ops.pow

reciprocal = ops.reciprocal

safediv = ops.safediv

safesub = ops.safesub

sigmoid = ops.sigmoid

sqrt = ops.sqrt

sub = ops.sub

tanh = ops.tanh

truediv = ops.truediv

xor = ops.xor

class AssociativeOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.Op`

class NullOp (*fn*, *, *name=None*)
Bases: `funsor.ops.AssociativeOp`

Placeholder associative op that unifies with any other op

class GetitemOp (*offset*)
Bases: `funsor.ops.op.Op`

Op encoding an index into one dimension, e.g. `x[:,:,y]` for offset of 2.

class AbsOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.UnaryOp`

class AddOp (*fn*, *, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class AndOp (*fn*, *, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class AtanhOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.TransformOp`

class EqOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.Op`

class ExpOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.TransformOp`

class GeOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.Op`

class GtOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.Op`

class InvertOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.UnaryOp`

class LeOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.Op`

class LgammaOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.UnaryOp`

class Log1pOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.UnaryOp`

class LogOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.TransformOp`

class LtOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.Op`

class MatmulOp (*fn*, *, *name=None*)
Bases: `funsor.ops.op.Op`

class MaxOp (*fn*, *, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class MinOp (*fn*, *, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class MulOp(*fn*, *, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class NeOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

class NegOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

class Op(*fn*, *, *name=None*)
Bases: `multipledispatch.dispatcher.Dispatcher`

classmethod subclass_register(**pattern*)

class OrOp(*fn*, *, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class PowOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

class ReciprocalOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.UnaryOp*

class SafedivOp(*fn*, *, *name=None*)
Bases: `funsor.ops.TruedivOp`

class SafesubOp(*fn*, *, *name=None*)
Bases: `funsor.ops.SubOp`

class SigmoidOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.TransformOp*

class SoftplusOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.UnaryOp*

class SqrtOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.UnaryOp*

class SubOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

class TanhOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.TransformOp*

class TransformOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.UnaryOp*

set_inv(*fn*)

Parameters fn (*callable*) – A function that inputs an arg `y` and outputs a value `x` such that `y=self(x)`.

set_log_abs_det_jacobian(*fn*)

Parameters fn (*callable*) – A function that inputs two args `x, y`, where `y=self(x)`, and returns `log(abs(det(dy/dx)))`.

static inv(*x*)

static log_abs_det_jacobian(*x*, *y*)

class TruedivOp(*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

class UnaryOp(*fn*, *, *name=None*)
Bases: `funsor.ops.op.Op`

class XorOp(*fn*, *, *name=None*)
Bases: `funsor.ops.AssociativeOp`

1.3 Array operations

all = ops.all

amax = ops.amax

amin = ops.amin

any = ops.any

astype = ops.astype

cat = ops.cat

cholesky = ops.cholesky

cholesky_inverse = ops.cholesky_inverse

cholesky_solve = ops.cholesky_solve

clamp = ops.clamp

detach = ops.detach

diagonal = ops.diagonal

einsum = ops.einsum

expand = ops.expand

finfo = ops.finfo

full_like = ops.full_like

is_numeric_array = ops.is_numeric_array

isnan = ops.isnan

logaddexp = ops.logaddexp

logsumexp = ops.logsumexp

new_arange = ops.new_arange

new_eye = ops.new_eye

new_zeros = ops.new_zeros

permute = ops.permute

prod = ops.prod

sample = ops.sample

stack = ops.stack

sum = ops.sum

transpose = ops.transpose

triangular_solve = ops.triangular_solve

unsqueeze = ops.unsqueeze

class ReshapeOp(*shape*)
Bases: *funsor.ops.op.Op*

class AddOp(*fn*, ***, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class AllOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class AmaxOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class AminOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class AnyOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class AssociativeOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class AstypeOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class AtanhOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.TransformOp*

class CatOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class ClampOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class DiagonalOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class EinsumOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class ExpOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.TransformOp*

class Full_likeOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class IsnanOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.Op*

class Log1pOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.UnaryOp*

class LogOp(*fn*, ***, *name=None*)
Bases: *funsor.ops.op.TransformOp*

class LogaddexpOp(*fn*, ***, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class MaxOp(*fn*, ***, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class MinOp(*fn*, ***, *name=None*)
Bases: `funsor.ops.AssociativeOp`

class Op (*fn*, *, *name=None*)
Bases: `multipledispatch.dispatcher.Dispatcher`

classmethod subclass_register (**pattern*)

class ProdOp (*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

class ReciprocalOp (*fn*, *, *name=None*)
Bases: *funsor.ops.op.UnaryOp*

class SafedivOp (*fn*, *, *name=None*)
Bases: `funsor.ops.TruedivOp`

class SafesubOp (*fn*, *, *name=None*)
Bases: `funsor.ops.SubOp`

class SampleOp (*fn*, *, *name=None*)
Bases: `funsor.ops.LogaddexpOp`

class SqrtOp (*fn*, *, *name=None*)
Bases: *funsor.ops.op.UnaryOp*

class StackOp (*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

class SumOp (*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

class TanhOp (*fn*, *, *name=None*)
Bases: *funsor.ops.op.TransformOp*

class TransposeOp (*fn*, *, *name=None*)
Bases: *funsor.ops.op.Op*

CHAPTER 2

Domains

Domain
alias of `builtins.type`

class BintType
Bases: `funsor.domains.ArrayType`

size

class RealsType
Bases: `funsor.domains.ArrayType`

dtype = 'real'

class Bint
Bases: `object`

Factory for bounded integer types:

```
Bint[5]           # integers ranging in {0,1,2,3,4}
Bint[2, 3, 3]     # 3x3 matrices with entries in {0,1}
```

dtype = None

shape = None

class Reals
Bases: `object`

Type of a real-valued array with known shape:

```
Reals[()] = Real   # scalar
Reals[8]           # vector of length 8
Reals[3, 3]        # 3x3 matrix
```

shape = None

class Real
Bases: `object`

shape = ()

reals(**args*)

bint(*size*)

find_domain(*op*, **domains*)

Finds the *Domain* resulting when applying `op` to `domains`. :param callable op: An operation. :param Domain *domains: One or more input domains.

CHAPTER 3

Interpretations

3.1 Interpreter

set_interpretation(*new*)

interpretation(*new*)

reinterpret(*x*)

Overloaded reinterpretation of a deferred expression.

This handles a limited class of expressions, raising `ValueError` in unhandled cases.

Parameters **x** (*A funsor or data structure holding funsors.*) – An input, typically involving deferred `Funsor` s.

Returns A reinterpreted version of the input.

Raises ValueError

dispatched_interpretation(*fn*)

Decorator to create a dispatched interpretation function.

class StatefulInterpretation

Bases: `object`

Base class for interpreters with instance-dependent state or parameters.

Example usage:

```
class MyInterpretation(StatefulInterpretation):

    def __init__(self, my_param):
        self.my_param = my_param

@MyInterpretation.register(...)
def my_impl(interpreter_state, cls, *args):
    my_param = interpreter_state.my_param
```

(continues on next page)

(continued from previous page)

```
    ...

with interpretation(MyInterpretation(my_param=0.1)):
    ...
```

classmethod dispatch(*key*, **args*)

classmethod register(**args*)

registry = <funsor.registry.KeyedRegistry object>

exception PatternMissingError
Bases: `NotImplementedError`

3.2 Monte Carlo

class MonteCarlo(*, *rng_key=None*, ***sample_inputs*)
Bases: *funsor.interpreter.StatefulInterpretation*

A Monte Carlo interpretation of *Integrate* expressions. This falls back to the previous interpreter in other cases.

Parameters rng_key –

classmethod dispatch(*key*, **args*)

registry = <funsor.registry.KeyedRegistry object>

3.3 Memoize

memoize(*cache=None*)
Exploit cons-hashing to do implicit common subexpression elimination

CHAPTER 4

Funsors

4.1 Basic Funsors

reflect (*cls*, **args*, ***kwargs*)
Construct a funsor, populate `._ast_values`, and cons hash. This is the only interpretation allowed to construct funsors.

lazy (*cls*, **args*)
Substitute eagerly but perform ops lazily.

eager (*cls*, **args*)
Eagerly execute ops with known implementations.

eager_or_die (*cls*, **args*)
Eagerly execute ops with known implementations. Disallows lazy `Subs` , `Unary` , `Binary` , and `Reduce` .

Raises `NotImplementedError` no pattern is found.

sequential (*cls*, **args*)
Eagerly execute ops with known implementations; additonally execute vectorized ops sequentially if no known vectorized implementation exists.

moment_matching (*cls*, **args*)
A moment matching interpretation of `Reduce` expressions. This falls back to `eager` in other cases.

class Funsor (*inputs*, *output*, *fresh=None*, *bound=None*)
Bases: `object`

Abstract base class for immutable functional tensors.

Concrete derived classes must implement `__init__()` methods taking hashable `*args` and no optional `**kwargs` so as to support cons hashing.

Derived classes with `.fresh` variables must implement an `eager_subs()` method. Derived classes with `.bound` variables must implement an `_alpha_convert()` method.

Parameters

- **inputs** (*OrderedDict*) – A mapping from input name to domain. This can be viewed as a typed context or a mapping from free variables to domains.
- **output** (*Domain*) – An output domain.

dtype

shape

input_vars

quote()

pretty(*maxlen=40*)

item()

requires_grad

reduce(*op*, *reduced_vars=None*)

Reduce along all or a subset of inputs.

Parameters

- **op** (*callable*) – A reduction operation.
- **reduced_vars** (*str, Variable, or set or frozenset thereof.*) – An optional input name or set of names to reduce. If unspecified, all inputs will be reduced.

sample(*sampled_vars*, *sample_inputs=None*, *rng_key=None*)

Create a Monte Carlo approximation to this funsor by replacing functions of `sampled_vars` with *Delta* s.

The result is a *Funsor* with the same `.inputs` and `.output` as the original funsor (plus `sample_inputs` if provided), so that self can be replaced by the sample in expectation computations:

```
y = x.sample(sampled_vars)
assert y.inputs == x.inputs
assert y.output == x.output
exact = (x.exp() * integrand).reduce(ops.add)
approx = (y.exp() * integrand).reduce(ops.add)
```

If `sample_inputs` is provided, this creates a batch of samples scaled samples.

Parameters

- **sampled_vars** (*str, Variable, or set or frozenset thereof.*) – A set of input variables to sample.
- **sample_inputs** (*OrderedDict*) – An optional mapping from variable name to *Domain* over which samples will be batched.
- **rng_key** (*None or JAX's random.PRNGKey*) – a PRNG state to be used by JAX backend to generate random samples

unscaled_sample(*sampled_vars*, *sample_inputs*, *rng_key=None*)

Internal method to draw an unscaled sample. This should be overridden by subclasses.

align(*names*)

Align this funsor to match given `names`. This is mainly useful in preparation for extracting `.data` of a *funsor.tensor.Tensor*.

Parameters **names** (*tuple*) – A tuple of strings representing all names but in a new order.

Returns A permuted funsor equivalent to self.

Return type *Funsor*

eager_subs(*subs*)
Internal substitution function. This relies on the user-facing `__call__()` method to coerce non-Funsors to Funsors. Once all inputs are Funsors, `eager_subs()` implementations can recurse to call `Subs`.

eager_unary(*op*)

eager_reduce(*op*, *reduced_vars*)

sequential_reduce(*op*, *reduced_vars*)

moment_matching_reduce(*op*, *reduced_vars*)

abs()

atanh()

sqrt()

exp()

log()

log1p()

sigmoid()

tanh()

reshape(*shape*)

sum()

prod()

logsumexp()

all()

any()

min()

max()

to_funsor(*x*, *output=None*, *dim_to_name=None*, ***kwargs*)
Convert to a `Funsor`. Only `Funsor` s and scalars are accepted.

Parameters

- **x** – An object.
- **output** (*funsor.domains.Domain*) – An optional output hint.
- **dim_to_name** (*OrderedDict*) – An optional mapping from negative batch dimensions to name strings.

Returns A Funsor equivalent to `x`.

Return type *Funsor*

Raises ValueError

to_data(*x*, *name_to_dim=None*, ***kwargs*)
Extract a python object from a `Funsor`.

Raises a `ValueError` if free variables remain or if the funsor is lazy.

Parameters

- **x** – An object, possibly a *Funsor*.
- **name_to_dim** (*OrderedDict*) – An optional inputs hint.

Returns A non-funsor equivalent to `x`.

Raises ValueError if any free variables remain.

Raises PatternMissingError if funsor is not fully evaluated.

class Variable(*name*, *output*)
Bases: *funsor.terms.Funsor*

Funsor representing a single free variable.

Parameters

- **name** (*str*) – A variable name.
- **output** (*funsor.domains.Domain*) – A domain.

eager_subs(*subs*)

class Subs(*arg*, *subs*)
Bases: *funsor.terms.Funsor*

Lazy substitution of the form `x(u=y, v=z)`.

Parameters

- **arg** (Funsor) – A funsor being substituted into.
- **subs** (*tuple*) – A tuple of `(name, value)` pairs, where `name` is a string and `value` can be coerced to a *Funsor* via *to_funsor()*.

unscaled_sample(*sampled_vars*, *sample_inputs*, *rng_key=None*)

class Unary(*op*, *arg*)
Bases: *funsor.terms.Funsor*

Lazy unary operation.

Parameters

- **op** (*Op*) – A unary operator.
- **arg** (Funsor) – An argument.

class Binary(*op*, *lhs*, *rhs*)
Bases: *funsor.terms.Funsor*

Lazy binary operation.

Parameters

- **op** (*Op*) – A binary operator.
- **lhs** (Funsor) – A left hand side argument.
- **rhs** (Funsor) – A right hand side argument.

class Reduce(*op*, *arg*, *reduced_vars*)
Bases: *funsor.terms.Funsor*

Lazy reduction over multiple variables.

Parameters

- **op** (*Op*) – A binary operator.

- **arg** (*funsor*) – An argument to be reduced.
- **reduced_vars** (*frozenset*) – A set of variables over which to reduce.

class Number (*data*, *dtype=None*)
Bases: *funsor.terms.Funsor*

Funsor backed by a Python number.

Parameters

- **data** (*numbers.Number*) – A python number.
- **dtype** – A nonnegative integer or the string "real".

item ()

eager_unary (*op*)

class Slice (*name*, *start*, *stop*, *step*, *dtype*)
Bases: *funsor.terms.Funsor*

Symbolic representation of a Python `slice` object.

Parameters

- **name** (*str*) – A name for the new slice dimension.
- **start** (*int*) –
- **stop** (*int*) –
- **step** (*int*) – Three args following `slice` semantics.
- **dtype** (*int*) – An optional bounded integer type of this slice.

eager_subs (*subs*)

class Stack (*name*, *parts*)
Bases: *funsor.terms.Funsor*

Stack of funsors along a new input dimension.

Parameters

- **name** (*str*) – The name of the new input variable along which to stack.
- **parts** (*tuple*) – A tuple of Funsors of homogenous output domain.

eager_subs (*subs*)

eager_reduce (*op*, *reduced_vars*)

class Cat (*name*, *parts*, *part_name=None*)
Bases: *funsor.terms.Funsor*

Concatenate funsors along an existing input dimension.

Parameters

- **name** (*str*) – The name of the input variable along which to concatenate.
- **parts** (*tuple*) – A tuple of Funsors of homogenous output domain.

eager_subs (*subs*)

class Lambda (*var*, *expr*)
Bases: *funsor.terms.Funsor*

Lazy inverse to `ops.getitem`.

This is useful to simulate higher-order functions of integers by representing those functions as arrays.

Parameters

- **var** (Variable) – A variable to bind.
- **expr** (*funsor*) – A funsor.

class Independent (*fn*, *reals_var*, *bint_var*, *diag_var*)

Bases: *funsor.terms.Funsor*

Creates an independent diagonal distribution.

This is equivalent to substitution followed by reduction:

```
f = ...  # a batched distribution
assert f.inputs['x_i'] == Reals[4, 5]
assert f.inputs['i'] == Bint[3]

g = Independent(f, 'x', 'i', 'x_i')
assert g.inputs['x'] == Reals[3, 4, 5]
assert 'x_i' not in g.inputs
assert 'i' not in g.inputs

x = Variable('x', Reals[3, 4, 5])
g == f(x_i=x['i']).reduce(ops.logaddexp, 'i')
```

Parameters

- **fn** (Funsor) – A funsor.
- **reals_var** (*str*) – The name of a real-tensor input.
- **bint_var** (*str*) – The name of a new batch input of `fn`.
- **diag_var** – The name of a smaller-shape real input of `fn`.

unscaled_sample (*sampled_vars*, *sample_inputs*, *rng_key=None*)

eager_subs (*subs*)

mean ()

variance ()

entropy ()

of_shape (**shape*)

4.2 Delta

solve (*expr*, *value*)

Tries to solve for free inputs of an `expr` such that `expr == value`, and computes the log-abs-det-Jacobian of the resulting substitution.

Parameters

- **expr** (Funsor) – An expression with a free variable.
- **value** (Funsor) – A target value.

Returns A tuple `(name, point, log_abs_det_jacobian)`

Return type tuple

Raises ValueError

class Delta(*terms*)
Bases: *funsor.terms.Funsor*

Normalized delta distribution binding multiple variables.

align(*names*)

eager_subs(*subs*)

eager_reduce(*op*, *reduced_vars*)

unscaled_sample(*sampled_vars*, *sample_inputs*, *rng_key=None*)

4.3 Tensor

ignore_jit_warnings()

class Tensor(*data*, *inputs=None*, *dtype='real'*)
Bases: *funsor.terms.Funsor*

Funsor backed by a PyTorch Tensor or a NumPy ndarray.

This follows the `torch.distributions` convention of arranging named "batch" dimensions on the left and remaining "event" dimensions on the right. The output shape is determined by all remaining dims. For example:

```
data = torch.zeros(5,4,3,2)
x = Tensor(data, OrderedDict([("i", Bint[5]), ("j", Bint[4])]))
assert x.output == Reals[3, 2]
```

Operators like `matmul` and `.sum()` operate only on the output shape, and will not change the named inputs.

Parameters

- **data** (*numeric_array*) – A PyTorch tensor or NumPy ndarray.
- **inputs** (*OrderedDict*) – An optional mapping from input name (str) to datatype (`funsor.domains.Domain`). Defaults to empty.
- **dtype** (*int or the string "real".*) – optional output datatype. Defaults to "real".

item()

clamp_finite()

requires_grad

align(*names*)

eager_subs(*subs*)

eager_unary(*op*)

eager_reduce(*op*, *reduced_vars*)

unscaled_sample(*sampled_vars*, *sample_inputs*, *rng_key=None*)

new_arange(*name*, **args*, ***kwargs*)
Helper to create a named `torch.arange()` or `np.arange()` funsor. In some cases this can be replaced by a symbolic *Slice*.

Parameters

- **name** (*str*) – A variable name.
- **start** (*int*) –
- **stop** (*int*) –
- **step** (*int*) – Three args following `slice` semantics.
- **dtype** (*int*) – An optional bounded integer type of this slice.

Return type *Tensor*

materialize(*x*)

Attempt to convert a Funsor to a `Number` or `Tensor` by substituting `arange()` s into its free variables.

Parameters **x** (`Funsor`) – A funsor.

Return type *Funsor*

align_tensor(*new_inputs*, *x*, *expand=False*)

Permute and add dims to a tensor to match desired `new_inputs`.

Parameters

- **new_inputs** (*OrderedDict*) – A target set of inputs.
- **x** (`funsor.terms.Funsor`) – A `Tensor` or `Number` .
- **expand** (*bool*) – If False (default), set result size to 1 for any input of `x` not in `new_inputs`; if True expand to `new_inputs` size.

Returns a number or `torch.Tensor` or `np.ndarray` that can be broadcast to other tensors with inputs `new_inputs`.

Return type int or float or torch.Tensor or np.ndarray

align_tensors(**args*, ***kwargs*)

Permute multiple tensors before applying a broadcasted op.

This is mainly useful for implementing eager funsor operations.

Parameters

- ***args** (`funsor.terms.Funsor`) – Multiple `Tensor` s and `Number` s.
- **expand** (*bool*) – Whether to expand input tensors. Defaults to False.

Returns a pair `(inputs, tensors)` where tensors are all `torch.Tensor` s or `np.ndarray` s that can be broadcast together to a single data with given `inputs`.

Return type tuple

class Function(*fn*, *output*, *args*)

Bases: `funsor.terms.Funsor`

Funsor wrapped by a native PyTorch or NumPy function.

Functions are assumed to support broadcasting and can be eagerly evaluated on funsors with free variables of int type (i.e. batch dimensions).

`Function` s are usually created via the `function()` decorator.

Parameters

- **fn** (*callable*) – A native PyTorch or NumPy function to wrap.
- **output** (*type*) – An output domain.

- **args** (Funsor) – Funsor arguments.

function (**signature*)

Decorator to wrap a PyTorch/NumPy function, using either type hints or explicit type annotations.

Example:

```
# Using type hints:
@funsor.tensor.function
def matmul(x: Reals[3, 4], y: Reals[4, 5]) -> Reals[3, 5]:
    return torch.matmul(x, y)

# Using explicit type annotations:
@funsor.tensor.function(Reals[3, 4], Reals[4, 5], Reals[3, 5])
def matmul(x, y):
    return torch.matmul(x, y)

@funsor.tensor.function(Reals[10], Reals[10, 10], Reals[10], Real)
def mvn_log_prob(loc, scale_tril, x):
    d = torch.distributions.MultivariateNormal(loc, scale_tril)
    return d.log_prob(x)
```

To support functions that output nested tuples of tensors, specify a nested `Tuple` of output types, for example:

```
@funsor.tensor.function
def max_and_argmax(x: Reals[8]) -> Tuple[Real, Bint[8]]:
    return torch.max(x, dim=-1)
```

Parameters ***signature** – A sequence if input domains followed by a final output domain or nested tuple of output domains.

class Einsum (*equation*, *operands*)

Bases: *funsor.terms.Funsor*

Wrapper around `torch.einsum()` or `np.einsum()` to operate on real-valued Funsors.

Note this operates only on the `output` tensor. To perform sum-product contractions on named dimensions, instead use + and *Reduce*.

Parameters

- **equation** (*str*) – An `torch.einsum()` or `np.einsum()` equation.
- **operands** (*tuple*) – A tuple of input funsors.

tensordot (*x*, *y*, *dims*)

Wrapper around `torch.tensordot()` or `np.tensordot()` to operate on real-valued Funsors.

Note this operates only on the `output` tensor. To perform sum-product contractions on named dimensions, instead use + and *Reduce*.

Arguments should satisfy:

```
len(x.shape) >= dims
len(y.shape) >= dims
dims == 0 or x.shape[-dims:] == y.shape[:dims]
```

Parameters

- **x** (Funsor) – A left hand argument.

- **y** (Funsor) – A y hand argument.
- **dims** (*int*) – The number of dimension of overlap of output shape.

Return type *Funsor*

stack(*parts*, *dim=0*)

Wrapper around `torch.stack()` or `np.stack()` to operate on real-valued Funsors.

Note this operates only on the `output` tensor. To stack funsors in a new named dim, instead use *Stack*.

Parameters

- **parts** (*tuple*) – A tuple of funsors.
- **dim** (*int*) – A torch dim along which to stack.

Return type *Funsor*

4.4 Gaussian

class BlockVector(*shape*)

Bases: `object`

Jit-compatible helper to build blockwise vectors. Syntax is similar to `torch.zeros()`

```
x = BlockVector((100, 20))
x[..., 0:4] = x1
x[..., 6:10] = x2
x = x.as_tensor()
assert x.shape == (100, 20)
```

as_tensor()

class BlockMatrix(*shape*)

Bases: `object`

Jit-compatible helper to build blockwise matrices. Syntax is similar to `torch.zeros()`

```
x = BlockMatrix((100, 20, 20))
x[..., 0:4, 0:4] = x11
x[..., 0:4, 6:10] = x12
x[..., 6:10, 0:4] = x12.transpose(-1, -2)
x[..., 6:10, 6:10] = x22
x = x.as_tensor()
assert x.shape == (100, 20, 20)
```

as_tensor()

align_gaussian(*new_inputs*, *old*)

Align data of a Gaussian distribution to a new `inputs` shape.

class Gaussian(*info_vec*, *precision*, *inputs*)

Bases: *funsor.terms.Funsor*

Funsor representing a batched joint Gaussian distribution as a log-density function.

Mathematically, a Gaussian represents the density function:

```
f(x) = < x | info_vec > - 0.5 * < x | precision | x >
     = < x | info_vec - 0.5 * precision @ x >
```

Note that *Gaussian* s are not normalized, rather they are canonicalized to evaluate to zero log density at the origin: `f(0) = 0`. This canonical form is useful in combination with the information filter representation because it allows *Gaussian* s with incomplete information, i.e. zero eigenvalues in the precision matrix. These incomplete distributions arise when making low-dimensional observations on higher dimensional hidden state.

Parameters

- **info_vec** (*torch.Tensor*) – An optional batched information vector, where `info_vec = precision @ mean`.
- **precision** (*torch.Tensor*) – A batched positive semidefinite precision matrix.
- **inputs** (*OrderedDict*) – Mapping from name to *Domain* .

log_normalizer

align (*names*)

eager_subs (*subs*)

eager_reduce (*op*, *reduced_vars*)

unscaled_sample (*sampled_vars*, *sample_inputs*, *rng_key=None*)

4.5 Joint

moment_matching_contract_default (**args*)

moment_matching_contract_joint (*red_op*, *bin_op*, *reduced_vars*, *discrete*, *gaussian*)

eager_reduce_exp (*op*, *arg*, *reduced_vars*)

eager_independent_joint (*joint*, *reals_var*, *bint_var*, *diag_var*)

4.6 Contraction

class Contraction (*red_op*, *bin_op*, *reduced_vars*, *terms*)

Bases: *funsor.terms.Funsor*

Declarative representation of a finitary sum-product operation.

After normalization via the `normalize()` interpretation contractions will canonically order their terms by type:

```
Delta, Number, Tensor, Gaussian
```

unscaled_sample (*sampled_vars*, *sample_inputs*, *rng_key=None*)

align (*names*)

GaussianMixture

alias of *funsor.cnf.Contraction*

recursion_reinterpret_contraction (*x*)

eager_contraction_generic_to_tuple (*red_op*, *bin_op*, *reduced_vars*, **terms*)

eager_contraction_generic_recursive (*red_op*, *bin_op*, *reduced_vars*, *terms*)

eager_contraction_to_reduce (*red_op*, *bin_op*, *reduced_vars*, *term*)

eager_contraction_to_binary (*red_op*, *bin_op*, *reduced_vars*, *lhs*, *rhs*)

eager_contraction_tensor (*red_op*, *bin_op*, *reduced_vars*, **terms*)

eager_contraction_gaussian (*red_op*, *bin_op*, *reduced_vars*, *x*, *y*)

normalize_contraction_commutative_canonical_order (*red_op*, *bin_op*, *reduced_vars*, **terms*)

normalize_contraction_commute_joint (*red_op*, *bin_op*, *reduced_vars*, *other*, *mixture*)

normalize_contraction_generic_args (*red_op*, *bin_op*, *reduced_vars*, **terms*)

normalize_trivial (*red_op*, *bin_op*, *reduced_vars*, *term*)

normalize_contraction_generic_tuple (*red_op*, *bin_op*, *reduced_vars*, *terms*)

binary_to_contract (*op*, *lhs*, *rhs*)

reduce_funsor (*op*, *arg*, *reduced_vars*)

unary_neg_variable (*op*, *arg*)

do_fresh_subs (*arg*, *subs*)

distribute_subs_contraction (*arg*, *subs*)

normalize_fuse_subs (*arg*, *subs*)

binary_subtract (*op*, *lhs*, *rhs*)

binary_divide (*op*, *lhs*, *rhs*)

unary_log_exp (*op*, *arg*)

unary_contract (*op*, *arg*)

4.7 Integrate

class Integrate (*log_measure*, *integrand*, *reduced_vars*)

Bases: `funsor.terms.Funsor`

Funsor representing an integral wrt a log density funsor.

Parameters

- **log_measure** (`Funsor`) – A log density funsor treated as a measure.
- **integrand** (`Funsor`) – An integrand funsor.
- **reduced_vars** (`str, Variable, or set or frozenset thereof.`) – An input name or set of names to reduce.

CHAPTER 5

Optimizer

unfold(*cls*, **args*)

unfold_contraction_generic_tuple(*red_op*, *bin_op*, *reduced_vars*, *terms*)

unfold_contraction_variadic(*r*, *b*, *v*, **ts*)

optimize(*cls*, **args*)

optimize_contraction_variadic(*r*, *b*, *v*, **ts*)

eager_contract_base(*red_op*, *bin_op*, *reduced_vars*, **terms*)

optimize_contract_finitary_funsor(*red_op*, *bin_op*, *reduced_vars*, *terms*)

apply_optimizer(*x*)

CHAPTER 6

Adjoint Algorithms

class AdjointTape
Bases: object

adjoint (*red_op*, *bin_op*, *root*, *targets*)

adjoint_tensor (*adj_redop*, *adj_binop*, *out_adj*, *data*, *inputs*, *dtype*)

adjoint_binary (*adj_redop*, *adj_binop*, *out_adj*, *op*, *lhs*, *rhs*)

adjoint_reduce (*adj_redop*, *adj_binop*, *out_adj*, *op*, *arg*, *reduced_vars*)

adjoint_contract_unary (*adj_redop*, *adj_binop*, *out_adj*, *sum_op*, *prod_op*, *reduced_vars*, *arg*)

adjoint_contract_generic (*adj_redop*, *adj_binop*, *out_adj*, *sum_op*, *prod_op*, *reduced_vars*, *terms*)

adjoint_contract (*adj_redop*, *adj_binop*, *out_adj*, *sum_op*, *prod_op*, *reduced_vars*, *lhs*, *rhs*)

adjoint_cat (*adj_redop*, *adj_binop*, *out_adj*, *name*, *parts*, *part_name*)

adjoint_subs_tensor (*adj_redop*, *adj_binop*, *out_adj*, *arg*, *subs*)

adjoint_subs_gaussianmixture_gaussianmixture (*adj_redop*, *adj_binop*, *out_adj*, *arg*, *subs*)

adjoint_subs_gaussian_gaussian (*adj_redop*, *adj_binop*, *out_adj*, *arg*, *subs*)

adjoint_subs_gaussianmixture_discrete (*adj_redop*, *adj_binop*, *out_adj*, *arg*, *subs*)

CHAPTER 7

Sum-Product Algorithms

partial_unroll (*factors*, *eliminate=frozenset()*, *plate_to_step={}*)

Performs partial unrolling of plated factor graphs to standard factor graphs. Only plates with history={0, 1} are supported.

For plates (history=0) unrolling operation appends `_{i}` suffix to variable names for index `i` in the plate (e.g., "x"->"x_0" for i=0). For markov dimensions (history=1) unrolling operation renames the suffixes `var_prev` to `var_{i}` and `var_curr` to `var_{i+1}` for index `i` (e.g., "x_prev"->"x_0" and "x_curr"->"x_1" for i=0). Markov vars are assumed to have names that follow `var_suffix` formatting and specifically `var_0` for the initial factor (e.g., `("x_0", "x_prev", "x_curr")` for history=1).

Parameters

- **factors** (`tuple or list`) – A collection of funsors.
- **eliminate** (`frozenset`) – A set of free variables to unroll, including both sum variables and product variable.
- **plate_to_step** (`dict`) – A dict mapping markov dimensions to `step` collections that contain ordered sequences of Markov variable names (e.g., `{"time": frozenset({("x_0", "x_prev", "x_curr")})}`). Plates are passed with an empty `step`.

Returns a list of partially unrolled Funsors, a frozenset of partially unrolled variable names, and a frozenset of remaining plates.

partial_sum_product (*sum_op*, *prod_op*, *factors*, *eliminate=frozenset()*, *plates=frozenset()*)

Performs partial sum-product contraction of a collection of factors.

Returns a list of partially contracted Funsors.

Return type list

modified_partial_sum_product (*sum_op*, *prod_op*, *factors*, *eliminate=frozenset()*, *plate_to_step={}*)

Generalization of the tensor variable elimination algorithm of `funsor.sum_product.partial_sum_product()` to handle markov dimensions in addition to plate dimensions. Markov dimensions in transition factors are eliminated efficiently using the parallel-scan algorithm in *funsor.sum_product.sequential_sum_product()*. The resulting factors are then combined with the initial factors and final states are eliminated. Therefore, when Markov dimension is eliminated `factors` has to contain a pairs of initial factors and transition factors.

Parameters

- **sum_op** (*AssociativeOp*) – A semiring sum operation.
- **prod_op** (*AssociativeOp*) – A semiring product operation.
- **factors** (*tuple or list*) – A collection of funsors.
- **eliminate** (*frozenset*) – A set of free variables to eliminate, including both sum variables and product variable.
- **plate_to_step** (*dict*) – A dict mapping markov dimensions to `step` collections that contain ordered sequences of Markov variable names (e.g., `{"time": frozenset({("x_0", "x_prev", "x_curr")})}`). Plates are passed with an empty `step`.

Returns a list of partially contracted Funsors.

Return type list

sum_product (*sum_op, prod_op, factors, eliminate=frozenset(), plates=frozenset()*)
Performs sum-product contraction of a collection of factors.

Returns a single contracted Funsor.

Return type *Funsor*

naive_sequential_sum_product (*sum_op, prod_op, trans, time, step*)

sequential_sum_product (*sum_op, prod_op, trans, time, step*)
For a funsor `trans` with dimensions `time`, `prev` and `curr`, computes a recursion equivalent to:

```
tail_time = 1 + arange("time", trans.inputs["time"].size - 1)
tail = sequential_sum_product(sum_op, prod_op,
                              trans(time=tail_time),
                              time, {"prev": "curr"})
return prod_op(trans(time=0)(curr="drop"), tail(prev="drop"))          .
↪reduce(sum_op, "drop")
```

but does so efficiently in parallel in O(log(time)).

Parameters

- **sum_op** (*AssociativeOp*) – A semiring sum operation.
- **prod_op** (*AssociativeOp*) – A semiring product operation.
- **trans** (`Funsor`) – A transition funsor.
- **time** (`Variable`) – The time input dimension.
- **step** (*dict*) – A dict mapping previous variables to current variables. This can contain multiple pairs of prev->curr variable names.

mixed_sequential_sum_product (*sum_op, prod_op, trans, time, step, num_segments=None*)
For a funsor `trans` with dimensions `time`, `prev` and `curr`, computes a recursion equivalent to:

```
tail_time = 1 + arange("time", trans.inputs["time"].size - 1)
tail = sequential_sum_product(sum_op, prod_op,
                              trans(time=tail_time),
                              time, {"prev": "curr"})
return prod_op(trans(time=0)(curr="drop"), tail(prev="drop"))          .
↪reduce(sum_op, "drop")
```

by mixing parallel and serial scan algorithms over `num_segments` segments.

Parameters

- **sum_op** (*AssociativeOp*) – A semiring sum operation.
- **prod_op** (*AssociativeOp*) – A semiring product operation.
- **trans** (`Funsor`) – A transition funsor.
- **time** (`Variable`) – The time input dimension.
- **step** (*dict*) – A dict mapping previous variables to current variables. This can contain multiple pairs of prev->curr variable names.
- **num_segments** (*int*) – number of segments for the first stage

naive_sarkka_bilmes_product (*sum_op*, *prod_op*, *trans*, *time_var*, *global_vars=frozenset()*)

sarkka_bilmes_product (*sum_op*, *prod_op*, *trans*, *time_var*, *global_vars=frozenset()*, *num_periods=1*)

class MarkovProductMeta (*name*, *bases*, *dct*)
Bases: `funsor.terms.FunsorMeta`

Wrapper to convert `step` to a tuple and fill in default `step_names`.

class MarkovProduct (*sum_op*, *prod_op*, *trans*, *time*, *step*, *step_names*)
Bases: *funsor.terms.Funsor*

Lazy representation of *sequential_sum_product()* .

Parameters

- **sum_op** (`AssociativeOp`) – A marginalization op.
- **prod_op** (`AssociativeOp`) – A Bayesian fusion op.
- **trans** (`Funsor`) – A sequence of transition factors, usually varying along the `time` input.
- **time** (*str or* `Variable`) – A time dimension.
- **step** (*dict*) – A str-to-str mapping of "previous" inputs of `trans` to "current" inputs of `trans`.
- **step_names** (*dict*) – Optional, for internal use by alpha conversion.

eager_subs (*subs*)

eager_markov_product (*sum_op*, *prod_op*, *trans*, *time*, *step*, *step_names*)

CHAPTER 8

Affine Pattern Matching

is_affine(*fn*)

A sound but incomplete test to determine whether a funsor is affine with respect to all of its real inputs.

Parameters fn (`Funsor`) – A funsor.

Return type bool

affine_inputs(*fn*)

Returns a [sound sub]set of real inputs of `fn` wrt which `fn` is known to be affine.

Parameters fn (`Funsor`) – A funsor.

Returns A set of input names wrt which `fn` is affine.

Return type frozenset

extract_affine(*fn*)

Extracts an affine representation of a funsor, satisfying:

```
x = ...
const, coeffs = extract_affine(x)
y = sum(Einsum(eqn, (coeff, Variable(var, coeff.output)))
        for var, (coeff, eqn) in coeffs.items())
assert_close(y, x)
assert frozenset(coeffs) == affine_inputs(x)
```

The `coeffs` will have one key per input wrt which `fn` is known to be affine (via *`affine_inputs()`*), and `const` and `coeffs.values` will all be constant wrt these inputs.

The affine approximation is computed by ev evaluating `fn` at zero and each basis vector. To improve performance, users may want to run under the *`memoize()`* interpretation.

Parameters fn (`Funsor`) – A funsor that is affine wrt the (add,mul) semiring in some subset of its inputs.

Returns A pair `(const, coeffs)` where const is a funsor with no real inputs and `coeffs` is an OrderedDict mapping input name to a `(coefficient, eqn)` pair in einsum form.

Return type tuple

CHAPTER 9

Testing Utiltites

xfail_if_not_implemented(*msg='Not implemented'*)

xfail_if_not_found(*msg='Not implemented'*)

class ActualExpected
Bases: `funsor.testing.LazyComparison`

Lazy string formatter for test assertions.

id_from_inputs(*inputs*)

is_array(*x*)

assert_close(*actual*, *expected*, *atol=1e-06*, *rtol=1e-06*)

check_funsor(*x*, *inputs*, *output*, *data=None*)
Check dims and shape modulo reordering.

xfail_param(**args*, ***kwargs*)

make_einsum_example(*equation*, *fill=None*, *sizes=(2, 3)*)

assert_equiv(*x*, *y*)
Check that two funsors are equivalent up to permutation of inputs.

rand(**args*)

randint(*low*, *high*, *size*)

randn(**args*)

random_scale_tril(**args*)

zeros(**args*)

ones(**args*)

empty(**args*)

random_tensor(*inputs*, *output=Real*)
Creates a random `funsor.tensor.Tensor` with given inputs and output.

random_gaussian(*inputs*)
Creates a random `funsor.gaussian.Gaussian` with given inputs.

random_mvn(*batch_shape*, *dim*, *diag=False*)
Generate a random `torch.distributions.MultivariateNormal` with given shape.

make_plated_hmm_einsum(*num_steps*, *num_obs_plates=1*, *num_hidden_plates=0*)

make_chain_einsum(*num_steps*)

make_hmm_einsum(*num_steps*)

CHAPTER 10

Pyro-Compatible Distributions

This interface provides a number of PyTorch-style distributions that use funsors internally to perform inference. These high-level objects are based on a wrapping class: `FunsorDistribution` which wraps a funsor in a PyTorch-distributions-compatible interface. *`FunsorDistribution`* objects can be used directly in Pyro models (using the standard Pyro backend).

10.1 FunsorDistribution Base Class

class FunsorDistribution(*funsor_dist*, *batch_shape=torch.Size([])*, *event_shape=torch.Size([])*, *dtype='real'*, *validate_args=None*)

Bases: `pyro.distributions.torch_distribution.TorchDistribution`

`Distribution` wrapper around a *`Funsor`* for use in Pyro code. This is typically used as a base class for specific funsor inference algorithms wrapped in a distribution interface.

Parameters

- **funsor_dist** (`funsor.terms.Funsor`) – A funsor with an input named "value" that is treated as a random variable. The distribution should be normalized over "value".
- **batch_shape** (*`torch.Size`*) – The distribution's batch shape. This must be in the same order as the input of the `funsor_dist`, but may contain extra dims of size 1.
- **event_shape** – The distribution's event shape.

arg_constraints = {}

support

log_prob(*value*)

sample(*sample_shape=torch.Size([])*)

rsample(*sample_shape=torch.Size([])*)

expand(*batch_shape*, *_instance=None*)

funsordistribution_to_funsor(*pyro_dist*, *output=None*, *dim_to_name=None*)

10.2 Hidden Markov Models

class DiscreteHMM(*initial_logits*, *transition_logits*, *observation_dist*, *validate_args=None*)

Bases: *funsor.pyro.distribution.FunsorDistribution*

Hidden Markov Model with discrete latent state and arbitrary observation distribution. This uses [1] to parallelize over time, achieving O(log(time)) parallel complexity.

The event_shape of this distribution includes time on the left:

```
event_shape = (num_steps,) + observation_dist.event_shape
```

This distribution supports any combination of homogeneous/heterogeneous time dependency of `transition_logits` and `observation_dist`. However, because time is included in this distribution's event_shape, the homogeneous+homogeneous case will have a broadcastable event_shape with `num_steps = 1`, allowing *log_prob()* to work with arbitrary length data:

```
# homogeneous + homogeneous case:
event_shape = (1,) + observation_dist.event_shape
```

This class should be interchangeable with `pyro.distributions.hmm.DiscreteHMM`.

References:

[1] Simo Sarkka, Angel F. Garcia-Fernandez (2019) "Temporal Parallelization of Bayesian Filters and Smoothers" https://arxiv.org/pdf/1905.13002.pdf

Parameters

- **initial_logits** (*Tensor*) – A logits tensor for an initial categorical distribution over latent states. Should have rightmost size `state_dim` and be broadcastable to `batch_shape + (state_dim,)`.
- **transition_logits** (*Tensor*) – A logits tensor for transition conditional distributions between latent states. Should have rightmost shape `(state_dim, state_dim)` (old, new), and be broadcastable to `batch_shape + (num_steps, state_dim, state_dim)`.
- **observation_dist** (*Distribution*) – A conditional distribution of observed data conditioned on latent state. The `.batch_shape` should have rightmost size `state_dim` and be broadcastable to `batch_shape + (num_steps, state_dim)`. The `.event_shape` may be arbitrary.

has_rsample

log_prob(*value*)

expand(*batch_shape*, *_instance=None*)

class GaussianHMM(*initial_dist*, *transition_matrix*, *transition_dist*, *observation_matrix*, *observation_dist*, *validate_args=None*)

Bases: *funsor.pyro.distribution.FunsorDistribution*

Hidden Markov Model with Gaussians for initial, transition, and observation distributions. This adapts [1] to parallelize over time to achieve O(log(time)) parallel complexity, however it differs in that it tracks the log normalizer to ensure `log_prob()` is differentiable.

This corresponds to the generative model:

```
z = initial_distribution.sample()
x = []
for t in range(num_steps):
    z = z @ transition_matrix + transition_dist.sample()
    x.append(z @ observation_matrix + observation_dist.sample())
```

The event_shape of this distribution includes time on the left:

```
event_shape = (num_steps,) + observation_dist.event_shape
```

This distribution supports any combination of homogeneous/heterogeneous time dependency of `transition_dist` and `observation_dist`. However, because time is included in this distribution's event_shape, the homogeneous+homogeneous case will have a broadcastable event_shape with `num_steps = 1`, allowing `log_prob()` to work with arbitrary length data:

```
event_shape = (1, obs_dim)  # homogeneous + homogeneous case
```

This class should be compatible with `pyro.distributions.hmm.GaussianHMM` , but additionally supports funsor *adjoint* algorithms.

References:

[1] Simo Sarkka, Angel F. Garcia-Fernandez (2019) "Temporal Parallelization of Bayesian Filters and Smoothers" https://arxiv.org/pdf/1905.13002.pdf

Variables

- **hidden_dim** (*int*) – The dimension of the hidden state.
- **obs_dim** (*int*) – The dimension of the observed state.

Parameters

- **initial_dist** (MultivariateNormal) – A distribution over initial states. This should have batch_shape broadcastable to `self.batch_shape`. This should have event_shape `(hidden_dim,)`.
- **transition_matrix** (*Tensor*) – A linear transformation of hidden state. This should have shape broadcastable to `self.batch_shape + (num_steps, hidden_dim, hidden_dim)` where the rightmost dims are ordered `(old, new)`.
- **transition_dist** (MultivariateNormal) – A process noise distribution. This should have batch_shape broadcastable to `self.batch_shape + (num_steps,)`. This should have event_shape `(hidden_dim,)`.
- **transition_matrix** – A linear transformation from hidden to observed state. This should have shape broadcastable to `self.batch_shape + (num_steps, hidden_dim, obs_dim)`.
- **observation_dist** (MultivariateNormal *or* Normal) – An observation noise distribution. This should have batch_shape broadcastable to `self.batch_shape + (num_steps,)`. This should have event_shape `(obs_dim,)`.

has_rsample = True

arg_constraints = {}

class GaussianMRF(*initial_dist*, *transition_dist*, *observation_dist*, *validate_args=None*)

Bases: *funsor.pyro.distribution.FunsorDistribution*

Temporal Markov Random Field with Gaussian factors for initial, transition, and observation distributions. This adapts [1] to parallelize over time to achieve O(log(time)) parallel complexity, however it differs in that it tracks the log normalizer to ensure `log_prob()` is differentiable.

The event_shape of this distribution includes time on the left:

```
event_shape = (num_steps,) + observation_dist.event_shape
```

This distribution supports any combination of homogeneous/heterogeneous time dependency of `transition_dist` and `observation_dist`. However, because time is included in this distribution's event_shape, the homogeneous+homogeneous case will have a broadcastable event_shape with `num_steps = 1`, allowing `log_prob()` to work with arbitrary length data:

```
event_shape = (1, obs_dim)  # homogeneous + homogeneous case
```

This class should be compatible with `pyro.distributions.hmm.GaussianMRF` , but additionally supports funsor *adjoint* algorithms.

References:

[1] Simo Sarkka, Angel F. Garcia-Fernandez (2019) "Temporal Parallelization of Bayesian Filters and Smoothers" https://arxiv.org/pdf/1905.13002.pdf

Variables

- **hidden_dim** (*int*) – The dimension of the hidden state.
- **obs_dim** (*int*) – The dimension of the observed state.

Parameters

- **initial_dist** (*MultivariateNormal*) – A distribution over initial states. This should have batch_shape broadcastable to `self.batch_shape`. This should have event_shape `(hidden_dim,)`.
- **transition_dist** (*MultivariateNormal*) – A joint distribution factor over a pair of successive time steps. This should have batch_shape broadcastable to `self.batch_shape + (num_steps,)`. This should have event_shape `(hidden_dim + hidden_dim,)` (old+new).
- **observation_dist** (*MultivariateNormal*) – A joint distribution factor over a hidden and an observed state. This should have batch_shape broadcastable to `self.batch_shape + (num_steps,)`. This should have event_shape `(hidden_dim + obs_dim,)`.

has_rsample = True

class SwitchingLinearHMM(*initial_logits, initial_mvn, transition_logits, transition_matrix, transition_mvn, observation_matrix, observation_mvn, exact=False, validate_args=None*)

Bases: *funsor.pyro.distribution.FunsorDistribution*

Switching Linear Dynamical System represented as a Hidden Markov Model.

This corresponds to the generative model:

```
z = Categorical(logits=initial_logits).sample()
y = initial_mvn[z].sample()
x = []
for t in range(num_steps):
```

(continues on next page)

(continued from previous page)

```
    z = Categorical(logits=transition_logits[t, z]).sample()
    y = y @ transition_matrix[t, z] + transition_mvn[t, z].sample()
    x.append(y @ observation_matrix[t, z] + observation_mvn[t, z].sample())
```

Viewed as a dynamic Bayesian network:

```
z[t-1] ----> z[t] ---> z[t+1]         Discrete latent class
   |  \        |  \        |   \
   | y[t-1] ----> y[t] ----> y[t+1]   Gaussian latent state
   |   /       |   /       |   /
   V  /        V  /        V  /
x[t-1]       x[t]      x[t+1]         Gaussian observation
```

Let `class` be the latent class, `state` be the latent multivariate normal state, and `value` be the observed multivariate normal value.

Parameters

- **initial_logits** (*Tensor*) – Represents `p(class[0])`.
- **initial_mvn** (MultivariateNormal) – Represents `p(state[0] | class[0])`.
- **transition_logits** (*Tensor*) – Represents `p(class[t+1] | class[t])`.
- **transition_matrix** (*Tensor*) –
- **transition_mvn** (MultivariateNormal) – Together with `transition_matrix`, this represents `p(state[t], state[t+1] | class[t])`.
- **observation_matrix** (*Tensor*) –
- **observation_mvn** (MultivariateNormal) – Together with `observation_matrix`, this represents `p(value[t+1], state[t+1] | class[t+1])`.
- **exact** (*bool*) – If True, perform exact inference at cost exponential in `num_steps`. If False, use a *moment_matching()* approximation and use parallel scan algorithm to reduce parallel complexity to logarithmic in `num_steps`. Defaults to False.

has_rsample = True

arg_constraints = {}

log_prob(*value*)

expand(*batch_shape*, *_instance=None*)

filter(*value*)

Compute posterior over final state given a sequence of observations.

Parameters value (*Tensor*) – A sequence of observations.

Returns A posterior distribution over latent states at the final time step, represented as a pair `(cat, mvn)`, where `Categorical` distribution over mixture components and `mvn` is a `MultivariateNormal` with rightmost batch dimension ranging over mixture components. This can then be used to initialize a sequential Pyro model for prediction.

Return type tuple

10.3 Conversion Utilities

This module follows a convention for converting between funsors and PyTorch distribution objects. This convention is compatible with NumPy/PyTorch-style broadcasting. Following PyTorch distributions (and Tensorflow distributions), we consider "event shapes" to be on the right and broadcast-compatible "batch shapes" to be on the left.

This module also aims to be forgiving in inputs and pedantic in outputs: methods accept either the superclass `torch.distributions.Distribution` objects or the subclass `pyro.distributions.TorchDistribution` objects. Methods return only the narrower subclass `pyro.distributions.TorchDistribution` objects.

tensor_to_funsor(*tensor*, *event_inputs=()*, *event_output=0*, *dtype='real'*)

Convert a `torch.Tensor` to a *`funsor.tensor.Tensor`*.

Note this should not touch data, but may trigger a `torch.Tensor.reshape()` op.

Parameters

- **tensor** (*torch.Tensor*) – A PyTorch tensor.
- **event_inputs** (*tuple*) – A tuple of names for rightmost tensor dimensions. If `tensor` has these names, they will be converted to `result.inputs`.
- **event_output** (*int*) – The number of tensor dimensions assigned to `result.output`. These must be on the right of any `event_input` dimensions.

Returns A funsor.

Return type *funsor.tensor.Tensor*

funsor_to_tensor(*funsor_*, *ndims*, *event_inputs=()*)

Convert a *`funsor.tensor.Tensor`* to a `torch.Tensor`.

Note this should not touch data, but may trigger a `torch.Tensor.reshape()` op.

Parameters

- **funsor** (`funsor.tensor.Tensor`) – A funsor.
- **ndims** (*int*) – The number of result dims, `== result.dim()`.
- **event_inputs** (*tuple*) – Names assigned to rightmost dimensions.

Returns A PyTorch tensor.

Return type torch.Tensor

dist_to_funsor(*pyro_dist*, *event_inputs=()*)

Convert a PyTorch distribution to a Funsor.

Parameters **torch.distribution.Distribution** – A PyTorch distribution.

Returns A funsor.

Return type *funsor.terms.Funsor*

mvn_to_funsor(*pyro_dist*, *event_inputs=()*, *real_inputs={}*)

Convert a joint `torch.distributions.MultivariateNormal` distribution into a *`Funsor`* with multiple real inputs.

This should satisfy:

```
sum(d.num_elements for d in real_inputs.values())
  == pyro_dist.event_shape[0]
```

Parameters

- **pyro_dist** (`torch.distributions.MultivariateNormal`) – A multivariate normal distribution over one or more variables of real or vector or tensor type.
- **event_inputs** (*tuple*) – A tuple of names for rightmost dimensions. These will be assigned to `result.inputs` of type `Bint`.
- **real_inputs** (*OrderedDict*) – A dict mapping real variable name to appropriately sized `Real`. The sum of all `.numel()` of all real inputs should be equal to the `pyro_dist` dimension.

Returns A funsor with given `real_inputs` and possibly additional Bint inputs.

Return type *funsor.terms.Funsor*

funsor_to_mvn(*gaussian*, *ndims*, *event_inputs=()*)

Convert a *Funsor* to a `pyro.distributions.MultivariateNormal` , dropping the normalization constant.

Parameters

- **gaussian** (`funsor.gaussian.Gaussian` *or* `funsor.joint.Joint`) – A Gaussian funsor.
- **ndims** (*int*) – The number of batch dimensions in the result.
- **event_inputs** (*tuple*) – A tuple of names to assign to rightmost dimensions.

Returns a multivariate normal distribution.

Return type pyro.distributions.MultivariateNormal

funsor_to_cat_and_mvn(*funsor_*, *ndims*, *event_inputs*)

Converts a labeled gaussian mixture model to a pair of distributions.

Parameters

- **funsor** (*funsor.joint.Joint*) – A Gaussian mixture funsor.
- **ndims** (*int*) – The number of batch dimensions in the result.

Returns A pair `(cat, mvn)`, where `cat` is a `Categorical` distribution over mixture components and `mvn` is a `MultivariateNormal` with rightmost batch dimension ranging over mixture components.

class AffineNormal(*matrix*, *loc*, *scale*, *value_x*, *value_y*)

Bases: *funsor.terms.Funsor*

Represents a conditional diagonal normal distribution over a random variable `Y` whose mean is an affine function of a random variable `X`. The likelihood of `X` is thus:

```
AffineNormal(matrix, loc, scale).condition(y).log_density(x)
```

which is equivalent to:

```
Normal(x @ matrix + loc, scale).to_event(1).log_prob(y)
```

Parameters

- **matrix** (`Funsor`) – A transformation from `X` to `Y`. Should have rightmost shape `(x_dim, y_dim)`.
- **loc** (`Funsor`) – A constant offset for `Y`'s mean. Should have output shape `(y_dim,)`.
- **scale** (`Funsor`) – Standard deviation for `Y`. Should have output shape `(y_dim,)`.

- **value_x** (Funsor) – A value X.
- **value_y** (Funsor) – A value Y.

matrix_and_mvn_to_funsor (*matrix*, *mvn*, *event_dims=()*, *x_name='value_x'*, *y_name='value_y'*)
Convert a noisy affine function to a Gaussian. The noisy affine function is defined as:

```
y = x @ matrix + mvn.sample()
```

The result is a non-normalized Gaussian funsor with two real inputs, `x_name` and `y_name`, corresponding to a conditional distribution of real vector `` y` given real vector ``x.

Parameters

- **matrix** (*torch.Tensor*) – A matrix with rightmost shape `(x_size, y_size)`.
- **mvn** (torch.distributions.MultivariateNormal *or torch.distributions.Independent of torch.distributions.Normal*) – A multivariate normal distribution with `event_shape == (y_size,)`.
- **event_dims** (*tuple*) – A tuple of names for rightmost dimensions. These will be assigned to `result.inputs` of type `Bint`.
- **x_name** (*str*) – The name of the `x` random variable.
- **y_name** (*str*) – The name of the `y` random variable.

Returns A funsor with given `real_inputs` and possibly additional Bint inputs.

Return type *funsor.terms.Funsor*

CHAPTER 11

Distribution Funsors

This interface provides a number of standard normalized probability distributions implemented as funsors.

class Distribution(**args*)
Bases: `funsor.terms.Funsor`

Funsor backed by a PyTorch/JAX distribution object.

Parameters ***args** – Distribution-dependent parameters. These can be either funsors or objects that can be coerced to funsors via `to_funsor()` . See derived classes for details.

dist_class = 'defined by derived classes'

eager_reduce(*op*, *reduced_vars*)

has_enumerate_support

classmethod eager_log_prob(**params*)

unscaled_sample(*sampled_vars*, *sample_inputs*, *rng_key=None*)

enumerate_support(*expand=False*)

entropy()

mean()

variance()

class Beta(*concentration1*, *concentration0*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Beta`

class Cauchy(*loc*, *scale*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Cauchy`

class Chi2(*df*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Chi2`

class BernoulliProbs(*probs*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `funsor.torch.distributions._PyroWrapper_BernoulliProbs`

class BernoulliLogits(*logits*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `funsor.torch.distributions._PyroWrapper_BernoulliLogits`

class Binomial(*total_count*, *probs*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Binomial`

class Categorical(*probs*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Categorical`

class CategoricalLogits(*logits*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `funsor.torch.distributions._PyroWrapper_CategoricalLogits`

class Delta(*v*, *log_density*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.delta.Delta`

class Dirichlet(*concentration*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Dirichlet`

class DirichletMultinomial(*concentration*, *total_count*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.conjugate.DirichletMultinomial`

class Exponential(*rate*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Exponential`

class Gamma(*concentration*, *rate*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Gamma`

class GammaPoisson(*concentration*, *rate*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.conjugate.GammaPoisson`

class Geometric(*probs*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.torch.Geometric`

class Gumbel(*loc*, *scale*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.torch.Gumbel`

class HalfCauchy(*scale*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.torch.HalfCauchy`

class HalfNormal(*scale*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.torch.HalfNormal`

class Laplace(*loc*, *scale*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.torch.Laplace`

class LowRankMultivariateNormal(*loc*, *cov_factor*, *cov_diag*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.torch.LowRankMultivariateNormal`

class Multinomial(*total_count*, *probs*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.torch.Multinomial`

class MultivariateNormal(*loc*, *scale_tril*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.torch.MultivariateNormal`

class NonreparameterizedBeta(*concentration1*, *concentration0*, *value='value'*)
Bases: *funsor.distribution.Distribution*

dist_class
alias of `pyro.distributions.testing.fakes.NonreparameterizedBeta`

class NonreparameterizedDirichlet(*concentration*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.testing.fakes.NonreparameterizedDirichlet`

class NonreparameterizedGamma(*concentration*, *rate*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.testing.fakes.NonreparameterizedGamma`

class NonreparameterizedNormal(*loc*, *scale*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.testing.fakes.NonreparameterizedNormal`

class Normal(*loc*, *scale*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Normal`

class Pareto(*scale*, *alpha*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Pareto`

class Poisson(*rate*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Poisson`

class StudentT(*df*, *loc*, *scale*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.StudentT`

class Uniform(*low*, *high*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.Uniform`

class VonMises(*loc*, *concentration*, *value='value'*)
Bases: `funsor.distribution.Distribution`

dist_class
alias of `pyro.distributions.torch.VonMises`

CHAPTER 12

Mini-Pyro Interface

This interface provides a backend for the Pyro probabilistic programming language. This interface is intended to be used indirectly by writing standard Pyro code and setting `pyro_backend("funsor")`. See examples/minipyro.py for example usage.

12.1 Mini Pyro

This file contains a minimal implementation of the Pyro Probabilistic Programming Language. The API (method signatures, etc.) match that of the full implementation as closely as possible. This file is independent of the rest of Pyro, with the exception of the `pyro.distributions` module.

An accompanying example that makes use of this implementation can be found at examples/minipyro.py.

class Distribution(*funsor_dist*, *sample_inputs=None*)

Bases: `object`

log_prob(*value*)

expand_inputs(*name*, *size*)

get_param_store()

class Messenger(*fn=None*)

Bases: `object`

process_message(*msg*)

postprocess_message(*msg*)

class trace(*fn=None*)

Bases: `funsor.minipyro.Messenger`

postprocess_message(*msg*)

get_trace(**args*, ***kwargs*)

class replay (*fn*, *guide_trace*)
Bases: `funsor.minipyro.Messenger`

process_message (*msg*)

class block (*fn=None*, *hide_fn=<function block.<lambda>>*)
Bases: `funsor.minipyro.Messenger`

process_message (*msg*)

class seed (*fn=None*, *rng_seed=None*)
Bases: `funsor.minipyro.Messenger`

class CondIndepStackFrame (*name*, *size*, *dim*)
Bases: `tuple`

dim
Alias for field number 2

name
Alias for field number 0

size
Alias for field number 1

class PlateMessenger (*fn*, *name*, *size*, *dim*)
Bases: `funsor.minipyro.Messenger`

process_message (*msg*)

tensor_to_funsor (*value*, *cond_indep_stack*, *output*)

class log_joint (*fn=None*)
Bases: `funsor.minipyro.Messenger`

process_message (*msg*)

postprocess_message (*msg*)

apply_stack (*msg*)

sample (*name*, *fn*, *obs=None*, *infer=None*)

param (*name*, *init_value=None*, *constraint=Real()*, *event_dim=None*)

plate (*name*, *size*, *dim*)

class PyroOptim (*optim_args*)
Bases: `object`

class Adam (*optim_args*)
Bases: `funsor.minipyro.PyroOptim`

TorchOptimizer
alias of `torch.optim.adam.Adam`

class ClippedAdam (*optim_args*)
Bases: `funsor.minipyro.PyroOptim`

TorchOptimizer
alias of `pyro.optim.clipped_adam.ClippedAdam`

class SVI (*model*, *guide*, *optim*, *loss*)
Bases: `object`

step (**args*, ***kwargs*)

Expectation(*log_probs*, *costs*, *sum_vars*, *prod_vars*)

elbo(*model*, *guide*, **args*, ***kwargs*)

class ELBO(***kwargs*)
Bases: `object`

class Trace_ELBO(***kwargs*)
Bases: *funsor.minipyro.ELBO*

class TraceMeanField_ELBO(***kwargs*)
Bases: *funsor.minipyro.ELBO*

class TraceEnum_ELBO(***kwargs*)
Bases: *funsor.minipyro.ELBO*

class Jit(*fn*, ***kwargs*)
Bases: `object`

class Jit_ELBO(*elbo*, ***kwargs*)
Bases: *funsor.minipyro.ELBO*

JitTrace_ELBO(***kwargs*)

JitTraceMeanField_ELBO(***kwargs*)

JitTraceEnum_ELBO(***kwargs*)

CHAPTER 13

Einsum Interface

This interface implements tensor variable elimination among tensors. In particular it does not implement continuous variable elimination.

naive_contract_einsum(*eqn*, **terms*, ***kwargs*)
Use for testing Contract against einsum

naive_einsum(*eqn*, **terms*, ***kwargs*)
Implements standard variable elimination.

naive_plated_einsum(*eqn*, **terms*, ***kwargs*)
Implements Tensor Variable Elimination (Algorithm 1 in [Obermeyer et al 2019])

[Obermeyer et al 2019] Obermeyer, F., Bingham, E., Jankowiak, M., Chiu, J., Pradhan, N., Rush, A., and Goodman, N. Tensor Variable Elimination for Plated Factor Graphs, 2019

einsum(*eqn*, **terms*, ***kwargs*)
Top-level interface for optimized tensor variable elimination.

Parameters

- **equation** (*str*) – An einsum equation.
- ***terms** (`funsor.terms.Funsor`) – One or more operands.
- **plates** (*set*) – Optional keyword argument denoting which funsor dimensions are plate dimensions. Among all input dimensions (from terms): dimensions in plates but not in outputs are product-reduced; dimensions in neither plates nor outputs are sum-reduced.

CHAPTER 14

Indices and tables

- genindex
- modindex
- search

Python Module Index

f

Index

A

B

F

G

H

I

J

L

M

N

O

P

Q

R

S

T

U

V

X

Z