
Funsor Documentation

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1	Operations	1
1.1	Operation classes	1
1.2	Builtin operations	2
1.3	Array operations	5
2	Domains	9
3	Interpretations	11
3.1	Interpreter	11
3.2	Monte Carlo	12
3.3	Memoize	12
4	Funsors	13
4.1	Basic Funsors	13
4.2	Delta	18
4.3	Tensor	19
4.4	Gaussian	22
4.5	Joint	23
4.6	Contraction	23
4.7	Integrate	24
5	Optimizer	25
6	Adjoint Algorithms	27
7	Sum-Product Algorithms	29
8	Affine Pattern Matching	33
9	Testing Utilites	35
10	Pyro-Compatible Distributions	37
10.1	FunsorDistribution Base Class	37
10.2	Hidden Markov Models	38
10.3	Conversion Utilities	42
11	Distribution Funsors	45

12 Mini-Pyro Interface	49
12.1 Mini Pyro	49
13 Einsum Interface	53
14 Indices and tables	55
Python Module Index	57
Index	59

1.1 Operation classes

class `CachedOpMeta`

Bases: `type`

Metaclass for caching op instance construction.

class `Op` (*fn*, *, *name=None*)

Bases: `multipledispatch.dispatcher.Dispatcher`

classmethod `subclass_register` (**pattern*)

make_op (*fn=None*, *parent=None*, *, *name=None*, *module_name='funsor.ops'*)

declare_op_types (*locals_*, *all_*, *name_*)

class `UnaryOp` (*fn*, *, *name=None*)

Bases: `funsor.ops.op.Op`

class `TransformOp` (*fn*, *, *name=None*)

Bases: `funsor.ops.op.UnaryOp`

set_inv (*fn*)

Parameters *fn* (*callable*) – A function that inputs an arg *y* and outputs a value *x* such that $y = \text{self}(x)$.

set_log_abs_det_jacobian (*fn*)

Parameters *fn* (*callable*) – A function that inputs two args *x*, *y*, where $y = \text{self}(x)$, and returns $\log(\text{abs}(\det(dy/dx)))$.

static inv (*x*)

static log_abs_det_jacobian (*x*, *y*)

1.2 Builtin operations

```
abs = ops.abs
add = ops.add
and_ = ops.and_
atanh = ops.atanh
eq = ops.eq
exp = ops.exp
ge = ops.ge
getitem = ops.GetItemOp(0)
    Op encoding an index into one dimension, e.g. x[:, :, y] for offset of 2.
gt = ops.gt
invert = ops.invert
le = ops.le
lgamma = ops.lgamma
log = ops.log
log1p = ops.log1p
lt = ops.lt
matmul = ops.matmul
max = ops.max
min = ops.min
mul = ops.mul
ne = ops.ne
neg = ops.neg
nullop = ops.nullop
    Placeholder associative op that unifies with any other op
or_ = ops.or_
pow = ops.pow
reciprocal = ops.reciprocal
safediv = ops.safediv
safesub = ops.safesub
sigmoid = ops.sigmoid
sqrt = ops.sqrt
sub = ops.sub
tanh = ops.tanh
truediv = ops.truediv
xor = ops.xor
```

```

class AssociativeOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class NullOp (fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

    Placeholder associative op that unifies with any other op

class GetitemOp (offset)
    Bases: funsor.ops.op.Op

    Op encoding an index into one dimension, e.g.  $x[:, :, y]$  for offset of 2.

class AbsOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class AddOp (fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

class AndOp (fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

class AtanhOp (fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class EqOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class ExpOp (fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class GeOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class GtOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class InvertOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class LeOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class LgammaOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class Log1pOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class LogOp (fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class LtOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class MatmulOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class MaxOp (fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

class MinOp (fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

```

```
class MulOp (fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

class NeOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class NegOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class Op (fn, *, name=None)
    Bases: multipledispatch.dispatcher.Dispatcher

    classmethod subclass_register (*pattern)

class OrOp (fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

class PowOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class ReciprocalOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class SafedivOp (fn, *, name=None)
    Bases: funsor.ops.TruedivOp

class SafesubOp (fn, *, name=None)
    Bases: funsor.ops.SubOp

class SigmoidOp (fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class SoftplusOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class SqrtOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class SubOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class TanhOp (fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class TransformOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

    set_inv (fn)

        Parameters fn (callable) – A function that inputs an arg y and outputs a value x such that
            y=self (x).

    set_log_abs_det_jacobian (fn)

        Parameters fn (callable) – A function that inputs two args x, y, where y=self (x), and
            returns log (abs (det (dy/dx) ) ).

    static inv (x)

    static log_abs_det_jacobian (x, y)

class TruedivOp (fn, *, name=None)
    Bases: funsor.ops.op.Op
```

```
class UnaryOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class XorOp(fn, *, name=None)
    Bases: funsor.ops.AssociativeOp
```

1.3 Array operations

```
all = ops.all
amax = ops.amax
amin = ops.amin
any = ops.any
astype = ops.astype
cat = ops.cat
cholesky = ops.cholesky
cholesky_inverse = ops.cholesky_inverse
cholesky_solve = ops.cholesky_solve
clamp = ops.clamp
detach = ops.detach
diagonal = ops.diagonal
einsum = ops.einsum
expand = ops.expand
finfo = ops.finfo
full_like = ops.full_like
is_numeric_array = ops.is_numeric_array
isnan = ops.isnan
logaddexp = ops.logaddexp
logsumexp = ops.logsumexp
new_arange = ops.new_arange
new_eye = ops.new_eye
new_zeros = ops.new_zeros
permute = ops.permute
prod = ops.prod
sample = ops.sample
stack = ops.stack
sum = ops.sum
transpose = ops.transpose
triangular_solve = ops.triangular_solve
```

```
unsqueeze = ops.unsqueeze

class ReshapeOp(shape)
    Bases: funsor.ops.op.Op

class AddOp(fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

class AllOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class AmaxOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class AminOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class AnyOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class AssociativeOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class AstypeOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class AtanhOp(fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class CatOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class ClampOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class DiagonalOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class EinsumOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class ExpOp(fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class Full_likeOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class IsnanOp(fn, *, name=None)
    Bases: funsor.ops.op.Op

class Log1pOp(fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class LogOp(fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class LogaddexpOp(fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

class MaxOp(fn, *, name=None)
    Bases: funsor.ops.AssociativeOp

class MinOp(fn, *, name=None)
    Bases: funsor.ops.AssociativeOp
```

```
class Op (fn, *, name=None)
    Bases: multipledispatch.dispatcher.Dispatcher

    classmethod subclass_register (*pattern)

class ProdOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class ReciprocalOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class SafedivOp (fn, *, name=None)
    Bases: funsor.ops.TruedivOp

class SafesubOp (fn, *, name=None)
    Bases: funsor.ops.SubOp

class SampleOp (fn, *, name=None)
    Bases: funsor.ops.LogaddexpOp

class SqrtOp (fn, *, name=None)
    Bases: funsor.ops.op.UnaryOp

class StackOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class SumOp (fn, *, name=None)
    Bases: funsor.ops.op.Op

class TanhOp (fn, *, name=None)
    Bases: funsor.ops.op.TransformOp

class TransposeOp (fn, *, name=None)
    Bases: funsor.ops.op.Op
```


CHAPTER 2

Domains

Domain

alias of `builtins.type`

class BintType

Bases: `funsor.domains.ArrayType`

size

class RealsType

Bases: `funsor.domains.ArrayType`

dtype = `'real'`

class Bint

Bases: `object`

Factory for bounded integer types:

```
Bint[5]           # integers ranging in {0,1,2,3,4}
Bint[2, 3, 3]     # 3x3 matrices with entries in {0,1}
```

dtype = `None`

shape = `None`

class Reals

Bases: `object`

Type of a real-valued array with known shape:

```
Reals[()] = Real  # scalar
Reals[8]       # vector of length 8
Reals[3, 3]    # 3x3 matrix
```

shape = `None`

class Real

Bases: `object`

```
    shape = ()  
reals (*args)  
bint (size)  
find_domain (op, *domains)  
    Finds the Domain resulting when applying op to domains. :param callable op: An operation. :param Domain  
    *domains: One or more input domains.
```

3.1 Interpreter

set_interpretation (*new*)

interpretation (*new*)

reinterpret (*x*)

Overloaded reinterpretation of a deferred expression.

This handles a limited class of expressions, raising `ValueError` in unhandled cases.

Parameters *x* (A *functor* or data structure holding *functors*.) – An input, typically involving deferred *Functors*.

Returns A reinterpreted version of the input.

Raises `ValueError`

dispatched_interpretation (*fn*)

Decorator to create a dispatched interpretation function.

class StatefulInterpretation

Bases: `object`

Base class for interpreters with instance-dependent state or parameters.

Example usage:

```
class MyInterpretation(StatefulInterpretation):

    def __init__(self, my_param):
        self.my_param = my_param

@MyInterpretation.register(...)
def my_impl(interpreter_state, cls, *args):
    my_param = interpreter_state.my_param
```

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```
...  
with interpretation(MyInterpretation(my_param=0.1)) :  
...
```

```
classmethod dispatch(key, *args)  
classmethod register(*args)  
registry = <funsor.registry.KeyedRegistry object>
```

```
exception PatternMissingError  
    Bases: NotImplementedError
```

3.2 Monte Carlo

```
class MonteCarlo(*, rng_key=None, **sample_inputs)  
    Bases: funsor.interpreter.StatefulInterpretation
```

A Monte Carlo interpretation of *Integrate* expressions. This falls back to the previous interpreter in other cases.

Parameters **rng_key** –

```
classmethod dispatch(key, *args)  
registry = <funsor.registry.KeyedRegistry object>
```

3.3 Memoize

```
memoize(cache=None)  
    Exploit cons-hashing to do implicit common subexpression elimination
```

4.1 Basic Funsors

reflect (*cls*, **args*, ***kwargs*)

Construct a funsor, populate `._ast_values`, and cons hash. This is the only interpretation allowed to construct funsors.

lazy (*cls*, **args*)

Substitute eagerly but perform ops lazily.

eager (*cls*, **args*)

Eagerly execute ops with known implementations.

eager_or_die (*cls*, **args*)

Eagerly execute ops with known implementations. Disallows lazy *Subs*, *Unary*, *Binary*, and *Reduce*.

Raises `NotImplementedError` no pattern is found.

sequential (*cls*, **args*)

Eagerly execute ops with known implementations; additionally execute vectorized ops sequentially if no known vectorized implementation exists.

moment_matching (*cls*, **args*)

A moment matching interpretation of *Reduce* expressions. This falls back to *eager* in other cases.

class Funsor (*inputs*, *output*, *fresh*=None, *bound*=None)

Bases: `object`

Abstract base class for immutable functional tensors.

Concrete derived classes must implement `__init__()` methods taking hashable **args* and no optional ***kwargs* so as to support cons hashing.

Derived classes with `.fresh` variables must implement an `eager_subs()` method. Derived classes with `.bound` variables must implement an `_alpha_convert()` method.

Parameters

- **inputs** (*OrderedDict*) – A mapping from input name to domain. This can be viewed as a typed context or a mapping from free variables to domains.
- **output** (*Domain*) – An output domain.

dtype

shape

input_vars

quote()

pretty (*maxlen=40*)

item()

requires_grad

reduce (*op, reduced_vars=None*)

Reduce along all or a subset of inputs.

Parameters

- **op** (*callable*) – A reduction operation.
- **reduced_vars** (*str, Variable, or set or frozenset thereof.*) – An optional input name or set of names to reduce. If unspecified, all inputs will be reduced.

sample (*sampled_vars, sample_inputs=None, rng_key=None*)

Create a Monte Carlo approximation to this funsor by replacing functions of *sampled_vars* with *Delta*s.

The result is a *Funsor* with the same *.inputs* and *.output* as the original funsor (plus *sample_inputs* if provided), so that self can be replaced by the sample in expectation computations:

```
y = x.sample(sampled_vars)
assert y.inputs == x.inputs
assert y.output == x.output
exact = (x.exp() * integrand).reduce(ops.add)
approx = (y.exp() * integrand).reduce(ops.add)
```

If *sample_inputs* is provided, this creates a batch of samples scaled samples.

Parameters

- **sampled_vars** (*str, Variable, or set or frozenset thereof.*) – A set of input variables to sample.
- **sample_inputs** (*OrderedDict*) – An optional mapping from variable name to *Domain* over which samples will be batched.
- **rng_key** (*None or JAX's random.PRNGKey*) – a PRNG state to be used by JAX backend to generate random samples

unscaled_sample (*sampled_vars, sample_inputs, rng_key=None*)

Internal method to draw an unscaled sample. This should be overridden by subclasses.

align (*names*)

Align this funsor to match given names. This is mainly useful in preparation for extracting *.data* of a *funsor.tensor.Tensor*.

Parameters **names** (*tuple*) – A tuple of strings representing all names but in a new order.

Returns A permuted funsor equivalent to self.

Return type *Funsor*

eager_subs (*subs*)

Internal substitution function. This relies on the user-facing `__call__()` method to coerce non-Funsors to Funsors. Once all inputs are Funsors, `eager_subs()` implementations can recurse to call *Subs*.

eager_unary (*op*)

eager_reduce (*op, reduced_vars*)

sequential_reduce (*op, reduced_vars*)

moment_matching_reduce (*op, reduced_vars*)

abs ()

atanh ()

sqrt ()

exp ()

log ()

log1p ()

sigmoid ()

tanh ()

reshape (*shape*)

sum ()

prod ()

logsumexp ()

all ()

any ()

min ()

max ()

to_funsor (*x, output=None, dim_to_name=None, **kwargs*)

Convert to a *Funsor*. Only *Funsor*s and scalars are accepted.

Parameters

- **x** – An object.
- **output** (*funsor.domains.Domain*) – An optional output hint.
- **dim_to_name** (*OrderedDict*) – An optional mapping from negative batch dimensions to name strings.

Returns A Funsor equivalent to *x*.

Return type *Funsor*

Raises `ValueError`

to_data (*x, name_to_dim=None, **kwargs*)

Extract a python object from a *Funsor*.

Raises a `ValueError` if free variables remain or if the funsor is lazy.

Parameters

- **x** – An object, possibly a *Funsor*.
- **name_to_dim** (*OrderedDict*) – An optional inputs hint.

Returns A non-funsor equivalent to *x*.

Raises *ValueError* if any free variables remain.

Raises *PatternMissingError* if funsor is not fully evaluated.

class Variable (*name, output*)

Bases: *funsor.terms.Funsor*

Funsor representing a single free variable.

Parameters

- **name** (*str*) – A variable name.
- **output** (*funsor.domains.Domain*) – A domain.

eager_subs (*subs*)

class Subs (*arg, subs*)

Bases: *funsor.terms.Funsor*

Lazy substitution of the form $x(u=y, v=z)$.

Parameters

- **arg** (*Funsor*) – A funsor being substituted into.
- **subs** (*tuple*) – A tuple of (*name*, *value*) pairs, where *name* is a string and *value* can be coerced to a *Funsor* via *to_funsor()*.

unscaled_sample (*sampled_vars, sample_inputs, rng_key=None*)

class Unary (*op, arg*)

Bases: *funsor.terms.Funsor*

Lazy unary operation.

Parameters

- **op** (*Op*) – A unary operator.
- **arg** (*Funsor*) – An argument.

class Binary (*op, lhs, rhs*)

Bases: *funsor.terms.Funsor*

Lazy binary operation.

Parameters

- **op** (*Op*) – A binary operator.
- **lhs** (*Funsor*) – A left hand side argument.
- **rhs** (*Funsor*) – A right hand side argument.

class Reduce (*op, arg, reduced_vars*)

Bases: *funsor.terms.Funsor*

Lazy reduction over multiple variables.

Parameters

- **op** (*Op*) – A binary operator.

- **arg** (*funsor*) – An argument to be reduced.
- **reduced_vars** (*frozenset*) – A set of variables over which to reduce.

class Number (*data, dtype=None*)
 Bases: *funsor.terms.Funsor*

Funsor backed by a Python number.

Parameters

- **data** (*numbers.Number*) – A python number.
- **dtype** – A nonnegative integer or the string “real”.

item ()

eager_unary (*op*)

class Slice (*name, start, stop, step, dtype*)
 Bases: *funsor.terms.Funsor*

Symbolic representation of a Python *slice* object.

Parameters

- **name** (*str*) – A name for the new slice dimension.
- **start** (*int*) –
- **stop** (*int*) –
- **step** (*int*) – Three args following *slice* semantics.
- **dtype** (*int*) – An optional bounded integer type of this slice.

eager_subs (*subs*)

class Stack (*name, parts*)
 Bases: *funsor.terms.Funsor*

Stack of funsors along a new input dimension.

Parameters

- **name** (*str*) – The name of the new input variable along which to stack.
- **parts** (*tuple*) – A tuple of Funsors of homogenous output domain.

eager_subs (*subs*)

eager_reduce (*op, reduced_vars*)

class Cat (*name, parts, part_name=None*)
 Bases: *funsor.terms.Funsor*

Concatenate funsors along an existing input dimension.

Parameters

- **name** (*str*) – The name of the input variable along which to concatenate.
- **parts** (*tuple*) – A tuple of Funsors of homogenous output domain.

eager_subs (*subs*)

class Lambda (*var, expr*)
 Bases: *funsor.terms.Funsor*

Lazy inverse to *ops.getitem*.

This is useful to simulate higher-order functions of integers by representing those functions as arrays.

Parameters

- **var** (*Variable*) – A variable to bind.
- **expr** (*funsor*) – A funsor.

class Independent (*fn, reals_var, bint_var, diag_var*)

Bases: *funsor.terms.Funsor*

Creates an independent diagonal distribution.

This is equivalent to substitution followed by reduction:

```
f = ... # a batched distribution
assert f.inputs['x_i'] == Reals[4, 5]
assert f.inputs['i'] == Bint[3]

g = Independent(f, 'x', 'i', 'x_i')
assert g.inputs['x'] == Reals[3, 4, 5]
assert 'x_i' not in g.inputs
assert 'i' not in g.inputs

x = Variable('x', Reals[3, 4, 5])
g == f(x_i=x['i']).reduce(ops.logaddexp, 'i')
```

Parameters

- **fn** (*Funsor*) – A funsor.
- **reals_var** (*str*) – The name of a real-tensor input.
- **bint_var** (*str*) – The name of a new batch input of *fn*.
- **diag_var** – The name of a smaller-shape real input of *fn*.

unscaled_sample (*sampled_vars, sample_inputs, rng_key=None*)

eager_subs (*subs*)

mean ()

variance ()

entropy ()

of_shape (**shape*)

4.2 Delta

solve (*expr, value*)

Tries to solve for free inputs of an *expr* such that *expr* == *value*, and computes the log-abs-det-Jacobian of the resulting substitution.

Parameters

- **expr** (*Funsor*) – An expression with a free variable.
- **value** (*Funsor*) – A target value.

Returns A tuple (*name, point, log_abs_det_jacobian*)

Return type `tuple`

Raises `ValueError`

class `Delta` (*terms*)

Bases: `funsor.terms.Funsor`

Normalized delta distribution binding multiple variables.

align (*names*)

eager_subs (*subs*)

eager_reduce (*op, reduced_vars*)

unscaled_sample (*sampled_vars, sample_inputs, rng_key=None*)

4.3 Tensor

ignore_jit_warnings ()

class `Tensor` (*data, inputs=None, dtype='real'*)

Bases: `funsor.terms.Funsor`

Funsor backed by a PyTorch Tensor or a NumPy ndarray.

This follows the `torch.distributions` convention of arranging named “batch” dimensions on the left and remaining “event” dimensions on the right. The output shape is determined by all remaining dims. For example:

```
data = torch.zeros(5,4,3,2)
x = Tensor(data, OrderedDict([("i", Bint[5]), ("j", Bint[4])]))
assert x.output == Reals[3, 2]
```

Operators like `matmul` and `.sum()` operate only on the output shape, and will not change the named inputs.

Parameters

- **data** (*numeric_array*) – A PyTorch tensor or NumPy ndarray.
- **inputs** (*OrderedDict*) – An optional mapping from input name (str) to datatype (`funsor.domains.Domain`). Defaults to empty.
- **dtype** (*int or the string "real".*) – optional output datatype. Defaults to “real”.

item ()

clamp_finite ()

requires_grad

align (*names*)

eager_subs (*subs*)

eager_unary (*op*)

eager_reduce (*op, reduced_vars*)

unscaled_sample (*sampled_vars, sample_inputs, rng_key=None*)

new_arange (*name, *args, **kwargs*)

Helper to create a named `torch.arange()` or `np.arange()` funsor. In some cases this can be replaced by a symbolic `Slice`.

Parameters

- **name** (*str*) – A variable name.
- **start** (*int*) –
- **stop** (*int*) –
- **step** (*int*) – Three args following *slice* semantics.
- **dtype** (*int*) – An optional bounded integer type of this slice.

Return type *Tensor***materialize** (*x*)

Attempt to convert a Funsor to a *Number* or *Tensor* by substituting *arange()* s into its free variables.

Parameters **x** (*Funsor*) – A funsor.**Return type** *Funsor***align_tensor** (*new_inputs, x, expand=False*)

Permute and add dims to a tensor to match desired *new_inputs*.

Parameters

- **new_inputs** (*OrderedDict*) – A target set of inputs.
- **x** (*funsor.terms.Funsor*) – A *Tensor* or *Number*.
- **expand** (*bool*) – If False (default), set result size to 1 for any input of *x* not in *new_inputs*; if True expand to *new_inputs* size.

Returns a number or *torch.Tensor* or *np.ndarray* that can be broadcast to other tensors with inputs *new_inputs*.

Return type *int* or *float* or *torch.Tensor* or *np.ndarray***align_tensors** (**args, **kwargs*)

Permute multiple tensors before applying a broadcasted op.

This is mainly useful for implementing eager funsor operations.

Parameters

- ***args** (*funsor.terms.Funsor*) – Multiple *Tensor* s and *Number* s.
- **expand** (*bool*) – Whether to expand input tensors. Defaults to False.

Returns a pair (*inputs, tensors*) where tensors are all *torch.Tensor* s or *np.ndarray* s that can be broadcast together to a single data with given inputs.

Return type *tuple***class Function** (*fn, output, args*)

Bases: *funsor.terms.Funsor*

Funsor wrapped by a native PyTorch or NumPy function.

Functions are assumed to support broadcasting and can be eagerly evaluated on funsors with free variables of *int* type (i.e. batch dimensions).

Function s are usually created via the *function()* decorator.

Parameters

- **fn** (*callable*) – A native PyTorch or NumPy function to wrap.
- **output** (*type*) – An output domain.

- **args** (*Funsor*) – Funsor arguments.

function (**signature*)

Decorator to wrap a PyTorch/NumPy function, using either type hints or explicit type annotations.

Example:

```
# Using type hints:
@funsor.tensor.function
def matmul(x: Reals[3, 4], y: Reals[4, 5]) -> Reals[3, 5]:
    return torch.matmul(x, y)

# Using explicit type annotations:
@funsor.tensor.function(Reals[3, 4], Reals[4, 5], Reals[3, 5])
def matmul(x, y):
    return torch.matmul(x, y)

@funsor.tensor.function(Reals[10], Reals[10, 10], Reals[10], Real)
def mvn_log_prob(loc, scale_tril, x):
    d = torch.distributions.MultivariateNormal(loc, scale_tril)
    return d.log_prob(x)
```

To support functions that output nested tuples of tensors, specify a nested `Tuple` of output types, for example:

```
@funsor.tensor.function
def max_and_argmax(x: Reals[8]) -> Tuple[Real, Bint[8]]:
    return torch.max(x, dim=-1)
```

Parameters **signature* – A sequence if input domains followed by a final output domain or nested tuple of output domains.

class `Einsum`(*equation*, *operands*)

Bases: `funsor.terms.Funsor`

Wrapper around `torch.einsum()` or `np.einsum()` to operate on real-valued Funsors.

Note this operates only on the output tensor. To perform sum-product contractions on named dimensions, instead use `+` and `Reduce`.

Parameters

- **equation** (*str*) – An `torch.einsum()` or `np.einsum()` equation.
- **operands** (*tuple*) – A tuple of input funsors.

tensordot (*x*, *y*, *dims*)

Wrapper around `torch.tensordot()` or `np.tensordot()` to operate on real-valued Funsors.

Note this operates only on the output tensor. To perform sum-product contractions on named dimensions, instead use `+` and `Reduce`.

Arguments should satisfy:

```
len(x.shape) >= dims
len(y.shape) >= dims
dims == 0 or x.shape[-dims:] == y.shape[:dims]
```

Parameters

- **x** (*Funsor*) – A left hand argument.

- **y** (`Funsor`) – A y hand argument.
- **dims** (`int`) – The number of dimension of overlap of output shape.

Return type *Funsor*

stack (*parts*, *dim=0*)

Wrapper around `torch.stack()` or `np.stack()` to operate on real-valued Funsors.

Note this operates only on the output tensor. To stack funsors in a new named dim, instead use *Stack*.

Parameters

- **parts** (*tuple*) – A tuple of funsors.
- **dim** (`int`) – A torch dim along which to stack.

Return type *Funsor*

4.4 Gaussian

class BlockVector (*shape*)

Bases: `object`

Jit-compatible helper to build blockwise vectors. Syntax is similar to `torch.zeros()`

```
x = BlockVector((100, 20))
x[..., 0:4] = x1
x[..., 6:10] = x2
x = x.as_tensor()
assert x.shape == (100, 20)
```

as_tensor ()

class BlockMatrix (*shape*)

Bases: `object`

Jit-compatible helper to build blockwise matrices. Syntax is similar to `torch.zeros()`

```
x = BlockMatrix((100, 20, 20))
x[..., 0:4, 0:4] = x11
x[..., 0:4, 6:10] = x12
x[..., 6:10, 0:4] = x12.transpose(-1, -2)
x[..., 6:10, 6:10] = x22
x = x.as_tensor()
assert x.shape == (100, 20, 20)
```

as_tensor ()

align_gaussian (*new_inputs*, *old*)

Align data of a Gaussian distribution to a new inputs shape.

class Gaussian (*info_vec*, *precision*, *inputs*)

Bases: *funsor.terms.Funsor*

Funsor representing a batched joint Gaussian distribution as a log-density function.

Mathematically, a Gaussian represents the density function:

```
f(x) = < x | info_vec > - 0.5 * < x | precision | x >
      = < x | info_vec - 0.5 * precision @ x >
```

Note that *Gaussian*s are not normalized, rather they are canonicalized to evaluate to zero log density at the origin: $f(0) = 0$. This canonical form is useful in combination with the information filter representation because it allows *Gaussian*s with incomplete information, i.e. zero eigenvalues in the precision matrix. These incomplete distributions arise when making low-dimensional observations on higher dimensional hidden state.

Parameters

- **info_vec** (*torch.Tensor*) – An optional batched information vector, where $\text{info_vec} = \text{precision} @ \text{mean}$.
- **precision** (*torch.Tensor*) – A batched positive semidefinite precision matrix.
- **inputs** (*OrderedDict*) – Mapping from name to *Domain*.

log_normalizer

align (*names*)

eager_subs (*subs*)

eager_reduce (*op, reduced_vars*)

unscaled_sample (*sampled_vars, sample_inputs, rng_key=None*)

4.5 Joint

moment_matching_contract_default (**args*)

moment_matching_contract_joint (*red_op, bin_op, reduced_vars, discrete, gaussian*)

eager_reduce_exp (*op, arg, reduced_vars*)

eager_independent_joint (*joint, reals_var, bint_var, diag_var*)

4.6 Contraction

class Contraction (*red_op, bin_op, reduced_vars, terms*)

Bases: *funsor.terms.Funsor*

Declarative representation of a finitary sum-product operation.

After normalization via the `normalize()` interpretation contractions will canonically order their terms by type:

Delta, Number, Tensor, Gaussian

unscaled_sample (*sampled_vars, sample_inputs, rng_key=None*)

align (*names*)

GaussianMixture

alias of *funsor.cnf.Contraction*

recursion_reinterpret_contraction (*x*)

eager_contraction_generic_to_tuple (*red_op, bin_op, reduced_vars, *terms*)

eager_contraction_generic_recursive (*red_op, bin_op, reduced_vars, terms*)

eager_contraction_to_reduce (*red_op, bin_op, reduced_vars, term*)

```
eager_contraction_to_binary(red_op, bin_op, reduced_vars, lhs, rhs)
eager_contraction_tensor(red_op, bin_op, reduced_vars, *terms)
eager_contraction_gaussian(red_op, bin_op, reduced_vars, x, y)
normalize_contraction_commutative_canonical_order(red_op, bin_op, reduced_vars,
                                                    *terms)
normalize_contraction_commute_joint(red_op, bin_op, reduced_vars, other, mixture)
normalize_contraction_generic_args(red_op, bin_op, reduced_vars, *terms)
normalize_trivial(red_op, bin_op, reduced_vars, term)
normalize_contraction_generic_tuple(red_op, bin_op, reduced_vars, terms)
binary_to_contract(op, lhs, rhs)
reduce_funsor(op, arg, reduced_vars)
unary_neg_variable(op, arg)
do_fresh_subs(arg, subs)
distribute_subs_contraction(arg, subs)
normalize_fuse_subs(arg, subs)
binary_subtract(op, lhs, rhs)
binary_divide(op, lhs, rhs)
unary_log_exp(op, arg)
unary_contract(op, arg)
```

4.7 Integrate

```
class Integrate(log_measure, integrand, reduced_vars)
```

Bases: `funsor.terms.Funsor`

Funsor representing an integral wrt a log density funsor.

Parameters

- **log_measure** (`Funsor`) – A log density funsor treated as a measure.
- **integrand** (`Funsor`) – An integrand funsor.
- **reduced_vars** (`str`, `Variable`, or `set` or `frozenset` thereof.) – An input name or set of names to reduce.

unfold(*cls*, **args*)

unfold_contraction_generic_tuple(*red_op*, *bin_op*, *reduced_vars*, *terms*)

unfold_contraction_variadic(*r*, *b*, *v*, **ts*)

optimize(*cls*, **args*)

optimize_contraction_variadic(*r*, *b*, *v*, **ts*)

eager_contract_base(*red_op*, *bin_op*, *reduced_vars*, **terms*)

optimize_contract_finitary_funsor(*red_op*, *bin_op*, *reduced_vars*, *terms*)

apply_optimizer(*x*)

Adjoint Algorithms

```
class AdjointTape
    Bases: object

    adjoint (red_op, bin_op, root, targets)
adjoint_tensor (adj_redop, adj_binop, out_adj, data, inputs, dtype)
adjoint_binary (adj_redop, adj_binop, out_adj, op, lhs, rhs)
adjoint_reduce (adj_redop, adj_binop, out_adj, op, arg, reduced_vars)
adjoint_contract_unary (adj_redop, adj_binop, out_adj, sum_op, prod_op, reduced_vars, arg)
adjoint_contract_generic (adj_redop, adj_binop, out_adj, sum_op, prod_op, reduced_vars, terms)
adjoint_contract (adj_redop, adj_binop, out_adj, sum_op, prod_op, reduced_vars, lhs, rhs)
adjoint_cat (adj_redop, adj_binop, out_adj, name, parts, part_name)
adjoint_subs_tensor (adj_redop, adj_binop, out_adj, arg, subs)
adjoint_subs_gaussianmixture_gaussianmixture (adj_redop, adj_binop, out_adj, arg, subs)
adjoint_subs_gaussian_gaussian (adj_redop, adj_binop, out_adj, arg, subs)
adjoint_subs_gaussianmixture_discrete (adj_redop, adj_binop, out_adj, arg, subs)
```


Sum-Product Algorithms

partial_unroll (*factors*, *eliminate=frozenset()*, *plate_to_step={}*)

Performs partial unrolling of plated factor graphs to standard factor graphs. Only plates with history={0, 1} are supported.

For plates (history=0) unrolling operation appends `_i` suffix to variable names for index `i` in the plate (e.g., “`x`”->“`x_0`” for `i=0`). For markov dimensions (history=1) unrolling operation renames the suffixes `var_prev` to `var_{i}` and `var_curr` to `var_{i+1}` for index `i` (e.g., “`x_prev`”->“`x_0`” and “`x_curr`”->“`x_1`” for `i=0`). Markov vars are assumed to have names that follow `var_suffix` formatting and specifically `var_0` for the initial factor (e.g., (“`x_0`”, “`x_prev`”, “`x_curr`”) for history=1).

Parameters

- **factors** (*tuple* or *list*) – A collection of funsors.
- **eliminate** (*frozenset*) – A set of free variables to unroll, including both sum variables and product variable.
- **plate_to_step** (*dict*) – A dict mapping markov dimensions to step collections that contain ordered sequences of Markov variable names (e.g., {"time": frozenset({"x_0", "x_prev", "x_curr"})}). Plates are passed with an empty step.

Returns a list of partially unrolled Funsors, a frozenset of partially unrolled variable names, and a frozenset of remaining plates.

partial_sum_product (*sum_op*, *prod_op*, *factors*, *eliminate=frozenset()*, *plates=frozenset()*)

Performs partial sum-product contraction of a collection of factors.

Returns a list of partially contracted Funsors.

Return type *list*

modified_partial_sum_product (*sum_op*, *prod_op*, *factors*, *eliminate=frozenset()*, *plate_to_step={}*)

Generalization of the tensor variable elimination algorithm of `funsor.sum_product.partial_sum_product()` to handle markov dimensions in addition to plate dimensions. Markov

dimensions in transition factors are eliminated efficiently using the parallel-scan algorithm in `funsor.sum_product.sequential_sum_product()`. The resulting factors are then combined with the initial factors and final states are eliminated. Therefore, when Markov dimension is eliminated factors has to contain a pairs of initial factors and transition factors.

Parameters

- **sum_op** (*AssociativeOp*) – A semiring sum operation.
- **prod_op** (*AssociativeOp*) – A semiring product operation.
- **factors** (*tuple or list*) – A collection of funsors.
- **eliminate** (*frozenset*) – A set of free variables to eliminate, including both sum variables and product variable.
- **plate_to_step** (*dict*) – A dict mapping markov dimensions to step collections that contain ordered sequences of Markov variable names (e.g., {"time": frozenset({"x_0", "x_prev", "x_curr"})}). Plates are passed with an empty step.

Returns a list of partially contracted Funsors.

Return type *list*

sum_product (*sum_op, prod_op, factors, eliminate=frozenset(), plates=frozenset()*)

Performs sum-product contraction of a collection of factors.

Returns a single contracted Funsor.

Return type *Funsor*

naive_sequential_sum_product (*sum_op, prod_op, trans, time, step*)

sequential_sum_product (*sum_op, prod_op, trans, time, step*)

For a funsor *trans* with dimensions *time*, *prev* and *curr*, computes a recursion equivalent to:

```
tail_time = 1 + arange("time", trans.inputs["time"].size - 1)
tail = sequential_sum_product(sum_op, prod_op,
                             trans(time=tail_time),
                             time, {"prev": "curr"})
return prod_op(trans(time=0)(curr="drop"), tail(prev="drop"))
→ reduce(sum_op, "drop")
```

but does so efficiently in parallel in $O(\log(\text{time}))$.

Parameters

- **sum_op** (*AssociativeOp*) – A semiring sum operation.
- **prod_op** (*AssociativeOp*) – A semiring product operation.
- **trans** (*Funsor*) – A transition funsor.
- **time** (*Variable*) – The time input dimension.
- **step** (*dict*) – A dict mapping previous variables to current variables. This can contain multiple pairs of prev->curr variable names.

mixed_sequential_sum_product (*sum_op, prod_op, trans, time, step, num_segments=None*)

For a funsor *trans* with dimensions *time*, *prev* and *curr*, computes a recursion equivalent to:

```

tail_time = 1 + arange("time", trans.inputs["time"].size - 1)
tail = sequential_sum_product(sum_op, prod_op,
                              trans(time=tail_time),
                              time, {"prev": "curr"})
return prod_op(trans(time=0)(curr="drop"), tail(prev="drop"))
↪ reduce(sum_op, "drop")

```

by mixing parallel and serial scan algorithms over `num_segments` segments.

Parameters

- **sum_op** (*AssociativeOp*) – A semiring sum operation.
- **prod_op** (*AssociativeOp*) – A semiring product operation.
- **trans** (*Funsor*) – A transition funsor.
- **time** (*Variable*) – The time input dimension.
- **step** (*dict*) – A dict mapping previous variables to current variables. This can contain multiple pairs of prev->curr variable names.
- **num_segments** (*int*) – number of segments for the first stage

naive_sarkka_bilmes_product (*sum_op, prod_op, trans, time_var, global_vars=frozenset()*)

sarkka_bilmes_product (*sum_op, prod_op, trans, time_var, global_vars=frozenset(), num_periods=1*)

class MarkovProductMeta (*name, bases, dct*)

Bases: `funsor.terms.FunsorMeta`

Wrapper to convert `step` to a tuple and fill in default `step_names`.

class MarkovProduct (*sum_op, prod_op, trans, time, step, step_names*)

Bases: `funsor.terms.Funsor`

Lazy representation of `sequential_sum_product()`.

Parameters

- **sum_op** (*AssociativeOp*) – A marginalization op.
- **prod_op** (*AssociativeOp*) – A Bayesian fusion op.
- **trans** (*Funsor*) – A sequence of transition factors, usually varying along the `time` input.
- **time** (*str* or *Variable*) – A time dimension.
- **step** (*dict*) – A str-to-str mapping of “previous” inputs of `trans` to “current” inputs of `trans`.
- **step_names** (*dict*) – Optional, for internal use by alpha conversion.

eager_subs (*subs*)

eager_markov_product (*sum_op, prod_op, trans, time, step, step_names*)

Affine Pattern Matching

is_affine (*fn*)

A sound but incomplete test to determine whether a funsor is affine with respect to all of its real inputs.

Parameters *fn* (`Funsor`) – A funsor.**Return type** `bool`**affine_inputs** (*fn*)Returns a [sound sub]set of real inputs of *fn* wrt which *fn* is known to be affine.**Parameters** *fn* (`Funsor`) – A funsor.**Returns** A set of input names wrt which *fn* is affine.**Return type** `frozenset`**extract_affine** (*fn*)

Extracts an affine representation of a funsor, satisfying:

```
x = ...
const, coeffs = extract_affine(x)
y = sum(Einsum(eqn, (coeff, Variable(var, coeff.output)))
        for var, (coeff, eqn) in coeffs.items())
assert_close(y, x)
assert frozenset(coeffs) == affine_inputs(x)
```

The `coeffs` will have one key per input wrt which *fn* is known to be affine (via `affine_inputs()`), and `const` and `coeffs.values` will all be constant wrt these inputs.

The affine approximation is computed by evaluating *fn* at zero and each basis vector. To improve performance, users may want to run under the `memoize()` interpretation.

Parameters *fn* (`Funsor`) – A funsor that is affine wrt the (add,mul) semiring in some subset of its inputs.

Returns A pair (`const`, `coeffs`) where `const` is a funsor with no real inputs and `coeffs` is an `OrderedDict` mapping input name to a (`coefficient`, `eqn`) pair in einsum form.

Return type `tuple`

```
xfail_if_not_implemented (msg='Not implemented')
xfail_if_not_found (msg='Not implemented')
class ActualExpected
    Bases: funsor.testing.LazyComparison
    Lazy string formatter for test assertions.
id_from_inputs (inputs)
is_array (x)
assert_close (actual, expected, atol=1e-06, rtol=1e-06)
check_funsor (x, inputs, output, data=None)
    Check dims and shape modulo reordering.
xfail_param (*args, **kwargs)
make_einsum_example (equation, fill=None, sizes=(2, 3))
assert_equiv (x, y)
    Check that two funsors are equivalent up to permutation of inputs.
rand (*args)
randint (low, high, size)
randn (*args)
random_scale_tril (*args)
zeros (*args)
ones (*args)
empty (*args)
random_tensor (inputs, output=Real)
    Creates a random funsor.tensor.Tensor with given inputs and output.
```

random_gaussian (*inputs*)

Creates a random *funsor.gaussian.Gaussian* with given inputs.

random_mvn (*batch_shape*, *dim*, *diag=False*)

Generate a random `torch.distributions.MultivariateNormal` with given shape.

make_plated_hmm_einsum (*num_steps*, *num_obs_plates=1*, *num_hidden_plates=0*)

make_chain_einsum (*num_steps*)

make_hmm_einsum (*num_steps*)

Pyro-Compatible Distributions

This interface provides a number of PyTorch-style distributions that use functors internally to perform inference. These high-level objects are based on a wrapping class: `FunctorDistribution` which wraps a functor in a PyTorch-distributions-compatible interface. `FunctorDistribution` objects can be used directly in Pyro models (using the standard Pyro backend).

10.1 FunctorDistribution Base Class

```
class FunctorDistribution (functor_dist, batch_shape=torch.Size([]), event_shape=torch.Size([]),  
                           dtype='real', validate_args=None)
```

Bases: `pyro.distributions.torch_distribution.TorchDistribution`

Distribution wrapper around a `Functor` for use in Pyro code. This is typically used as a base class for specific functor inference algorithms wrapped in a distribution interface.

Parameters

- **functor_dist** (`functor.terms.Functor`) – A functor with an input named “value” that is treated as a random variable. The distribution should be normalized over “value”.
- **batch_shape** (`torch.Size`) – The distribution’s batch shape. This must be in the same order as the input of the `functor_dist`, but may contain extra dims of size 1.
- **event_shape** – The distribution’s event shape.

```
arg_constraints = {}
```

```
support
```

```
log_prob (value)
```

```
sample (sample_shape=torch.Size([]))
```

```
rsample (sample_shape=torch.Size([]))
```

```
expand (batch_shape, _instance=None)
```

```
functordistribution_to_functor (pyro_dist, output=None, dim_to_name=None)
```

10.2 Hidden Markov Models

class DiscreteHMM(*initial_logits, transition_logits, observation_dist, validate_args=None*)

Bases: *funsor.pyro.distribution.FunsorDistribution*

Hidden Markov Model with discrete latent state and arbitrary observation distribution. This uses [1] to parallelize over time, achieving $O(\log(\text{time}))$ parallel complexity.

The event_shape of this distribution includes time on the left:

```
event_shape = (num_steps,) + observation_dist.event_shape
```

This distribution supports any combination of homogeneous/heterogeneous time dependency of *transition_logits* and *observation_dist*. However, because time is included in this distribution's event_shape, the homogeneous+homogeneous case will have a broadcastable event_shape with *num_steps* = 1, allowing *log_prob()* to work with arbitrary length data:

```
# homogeneous + homogeneous case:
event_shape = (1,) + observation_dist.event_shape
```

This class should be interchangeable with `pyro.distributions.hmm.DiscreteHMM`.

References:

[1] **Simo Sarkka, Angel F. Garcia-Fernandez (2019)** “Temporal Parallelization of Bayesian Filters and Smoothers” <https://arxiv.org/pdf/1905.13002.pdf>

Parameters

- **initial_logits** (*Tensor*) – A logits tensor for an initial categorical distribution over latent states. Should have rightmost size *state_dim* and be broadcastable to *batch_shape* + (*state_dim*,).
- **transition_logits** (*Tensor*) – A logits tensor for transition conditional distributions between latent states. Should have rightmost shape (*state_dim*, *state_dim*) (old, new), and be broadcastable to *batch_shape* + (*num_steps*, *state_dim*, *state_dim*).
- **observation_dist** (*Distribution*) – A conditional distribution of observed data conditioned on latent state. The *.batch_shape* should have rightmost size *state_dim* and be broadcastable to *batch_shape* + (*num_steps*, *state_dim*). The *.event_shape* may be arbitrary.

has_rsample

log_prob (*value*)

expand (*batch_shape, _instance=None*)

class GaussianHMM(*initial_dist, transition_matrix, transition_dist, observation_matrix, observation_dist, validate_args=None*)

Bases: *funsor.pyro.distribution.FunsorDistribution*

Hidden Markov Model with Gaussians for initial, transition, and observation distributions. This adapts [1] to parallelize over time to achieve $O(\log(\text{time}))$ parallel complexity, however it differs in that it tracks the log normalizer to ensure *log_prob()* is differentiable.

This corresponds to the generative model:

```

z = initial_distribution.sample()
x = []
for t in range(num_steps):
    z = z @ transition_matrix + transition_dist.sample()
    x.append(z @ observation_matrix + observation_dist.sample())

```

The `event_shape` of this distribution includes time on the left:

```
event_shape = (num_steps,) + observation_dist.event_shape
```

This distribution supports any combination of homogeneous/heterogeneous time dependency of `transition_dist` and `observation_dist`. However, because time is included in this distribution's `event_shape`, the homogeneous+homogeneous case will have a broadcastable `event_shape` with `num_steps = 1`, allowing `log_prob()` to work with arbitrary length data:

```
event_shape = (1, obs_dim) # homogeneous + homogeneous case
```

This class should be compatible with `pyro.distributions.hmm.GaussianHMM`, but additionally supports funsor *adjoint* algorithms.

References:

[1] Simo Sarkka, Angel F. Garcia-Fernandez (2019) “Temporal Parallelization of Bayesian Filters and Smoothers” <https://arxiv.org/pdf/1905.13002.pdf>

Variables

- **hidden_dim** (*int*) – The dimension of the hidden state.
- **obs_dim** (*int*) – The dimension of the observed state.

Parameters

- **initial_dist** (*MultivariateNormal*) – A distribution over initial states. This should have `batch_shape` broadcastable to `self.batch_shape`. This should have `event_shape (hidden_dim,)`.
- **transition_matrix** (*Tensor*) – A linear transformation of hidden state. This should have shape broadcastable to `self.batch_shape + (num_steps, hidden_dim, hidden_dim)` where the rightmost dims are ordered (old, new).
- **transition_dist** (*MultivariateNormal*) – A process noise distribution. This should have `batch_shape` broadcastable to `self.batch_shape + (num_steps,)`. This should have `event_shape (hidden_dim,)`.
- **transition_matrix** – A linear transformation from hidden to observed state. This should have shape broadcastable to `self.batch_shape + (num_steps, hidden_dim, obs_dim)`.
- **observation_dist** (*MultivariateNormal or Normal*) – An observation noise distribution. This should have `batch_shape` broadcastable to `self.batch_shape + (num_steps,)`. This should have `event_shape (obs_dim,)`.

has_rsample = True

arg_constraints = {}

```

class GaussianMRF (initial_dist, transition_dist, observation_dist, validate_args=None)
    Bases: funsor.pyro.distribution.FunsorDistribution

```

Temporal Markov Random Field with Gaussian factors for initial, transition, and observation distributions. This adapts [1] to parallelize over time to achieve $O(\log(\text{time}))$ parallel complexity, however it differs in that it tracks the log normalizer to ensure `log_prob()` is differentiable.

The `event_shape` of this distribution includes time on the left:

```
event_shape = (num_steps,) + observation_dist.event_shape
```

This distribution supports any combination of homogeneous/heterogeneous time dependency of `transition_dist` and `observation_dist`. However, because time is included in this distribution's `event_shape`, the homogeneous+homogeneous case will have a broadcastable `event_shape` with `num_steps = 1`, allowing `log_prob()` to work with arbitrary length data:

```
event_shape = (1, obs_dim) # homogeneous + homogeneous case
```

This class should be compatible with `pyro.distributions.hmm.GaussianMRF`, but additionally supports funsor *adjoint* algorithms.

References:

[1] Simo Sarkka, Angel F. Garcia-Fernandez (2019) “Temporal Parallelization of Bayesian Filters and Smoothers” <https://arxiv.org/pdf/1905.13002.pdf>

Variables

- **hidden_dim** (*int*) – The dimension of the hidden state.
- **obs_dim** (*int*) – The dimension of the observed state.

Parameters

- **initial_dist** (`MultivariateNormal`) – A distribution over initial states. This should have `batch_shape` broadcastable to `self.batch_shape`. This should have `event_shape (hidden_dim,)`.
- **transition_dist** (`MultivariateNormal`) – A joint distribution factor over a pair of successive time steps. This should have `batch_shape` broadcastable to `self.batch_shape + (num_steps,)`. This should have `event_shape (hidden_dim + hidden_dim,)` (old+new).
- **observation_dist** (`MultivariateNormal`) – A joint distribution factor over a hidden and an observed state. This should have `batch_shape` broadcastable to `self.batch_shape + (num_steps,)`. This should have `event_shape (hidden_dim + obs_dim,)`.

has_rsample = True

```
class SwitchingLinearHMM(initial_logits, initial_mvn, transition_logits, transition_matrix, transition_mvn, observation_matrix, observation_mvn, exact=False, validate_args=None)
```

Bases: `funsor.pyro.distribution.FunsorDistribution`

Switching Linear Dynamical System represented as a Hidden Markov Model.

This corresponds to the generative model:

```
z = Categorical(logits=initial_logits).sample()
y = initial_mvn[z].sample()
x = []
for t in range(num_steps):
```

(continues on next page)

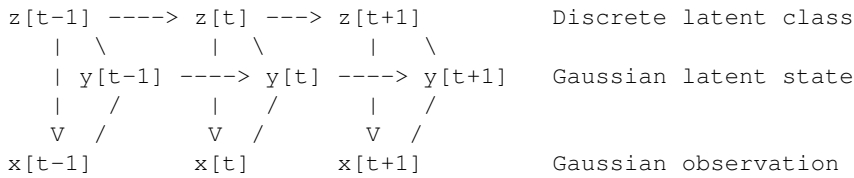
(continued from previous page)

```

z = Categorical(logits=transition_logits[t, z]).sample()
y = y @ transition_matrix[t, z] + transition_mvn[t, z].sample()
x.append(y @ observation_matrix[t, z] + observation_mvn[t, z].sample())

```

Viewed as a dynamic Bayesian network:



Let `class` be the latent class, `state` be the latent multivariate normal state, and `value` be the observed multivariate normal value.

Parameters

- **initial_logits** (*Tensor*) – Represents $p(\text{class}[0])$.
- **initial_mvn** (*MultivariateNormal*) – Represents $p(\text{state}[0] \mid \text{class}[0])$.
- **transition_logits** (*Tensor*) – Represents $p(\text{class}[t+1] \mid \text{class}[t])$.
- **transition_matrix** (*Tensor*) –
- **transition_mvn** (*MultivariateNormal*) – Together with `transition_matrix`, this represents $p(\text{state}[t], \text{state}[t+1] \mid \text{class}[t])$.
- **observation_matrix** (*Tensor*) –
- **observation_mvn** (*MultivariateNormal*) – Together with `observation_matrix`, this represents $p(\text{value}[t+1], \text{state}[t+1] \mid \text{class}[t+1])$.
- **exact** (*bool*) – If `True`, perform exact inference at cost exponential in `num_steps`. If `False`, use a *moment_matching()* approximation and use parallel scan algorithm to reduce parallel complexity to logarithmic in `num_steps`. Defaults to `False`.

has_rsample = `True`

arg_constraints = `{}`

log_prob (*value*)

expand (*batch_shape*, *_instance=None*)

filter (*value*)

Compute posterior over final state given a sequence of observations.

Parameters **value** (*Tensor*) – A sequence of observations.

Returns A posterior distribution over latent states at the final time step, represented as a pair (`cat`, `mvn`), where `Categorical` distribution over mixture components and `mvn` is a `MultivariateNormal` with rightmost batch dimension ranging over mixture components. This can then be used to initialize a sequential Pyro model for prediction.

Return type *tuple*

10.3 Conversion Utilities

This module follows a convention for converting between funsors and PyTorch distribution objects. This convention is compatible with NumPy/PyTorch-style broadcasting. Following PyTorch distributions (and Tensorflow distributions), we consider “event shapes” to be on the right and broadcast-compatible “batch shapes” to be on the left.

This module also aims to be forgiving in inputs and pedantic in outputs: methods accept either the superclass `torch.distributions.Distribution` objects or the subclass `pyro.distributions.TorchDistribution` objects. Methods return only the narrower subclass `pyro.distributions.TorchDistribution` objects.

tensor_to_funsor (*tensor*, *event_inputs*=(), *event_output*=0, *dtype*='real')

Convert a `torch.Tensor` to a `funsor.tensor.Tensor`.

Note this should not touch data, but may trigger a `torch.Tensor.reshape()` op.

Parameters

- **tensor** (`torch.Tensor`) – A PyTorch tensor.
- **event_inputs** (`tuple`) – A tuple of names for rightmost tensor dimensions. If `tensor` has these names, they will be converted to `result.inputs`.
- **event_output** (`int`) – The number of tensor dimensions assigned to `result.output`. These must be on the right of any `event_input` dimensions.

Returns A funsor.

Return type `funsor.tensor.Tensor`

funsor_to_tensor (*funsor_*, *ndims*, *event_inputs*=())

Convert a `funsor.tensor.Tensor` to a `torch.Tensor`.

Note this should not touch data, but may trigger a `torch.Tensor.reshape()` op.

Parameters

- **funsor** (`funsor.tensor.Tensor`) – A funsor.
- **ndims** (`int`) – The number of result dims, == `result.dim()`.
- **event_inputs** (`tuple`) – Names assigned to rightmost dimensions.

Returns A PyTorch tensor.

Return type `torch.Tensor`

dist_to_funsor (*pyro_dist*, *event_inputs*=())

Convert a PyTorch distribution to a Funsor.

Parameters `torch.distribution.Distribution` – A PyTorch distribution.

Returns A funsor.

Return type `funsor.terms.Funsor`

mvn_to_funsor (*pyro_dist*, *event_inputs*=(), *real_inputs*={})

Convert a joint `torch.distributions.MultivariateNormal` distribution into a `Funsor` with multiple real inputs.

This should satisfy:

```
sum(d.num_elements for d in real_inputs.values())
== pyro_dist.event_shape[0]
```

Parameters

- **pyro_dist** (`torch.distributions.MultivariateNormal`) – A multivariate normal distribution over one or more variables of real or vector or tensor type.
- **event_inputs** (`tuple`) – A tuple of names for rightmost dimensions. These will be assigned to `result.inputs` of type `Bint`.
- **real_inputs** (`OrderedDict`) – A dict mapping real variable name to appropriately sized `Real`. The sum of all `.numel()` of all real inputs should be equal to the `pyro_dist` dimension.

Returns A funsor with given `real_inputs` and possibly additional `Bint` inputs.

Return type *funsor.terms.Funsor*

funsor_to_mvn (*gaussian*, *ndims*, *event_inputs*=())

Convert a *Funsor* to a `pyro.distributions.MultivariateNormal`, dropping the normalization constant.

Parameters

- **gaussian** (`funsor.gaussian.Gaussian` or `funsor.joint.Joint`) – A Gaussian funsor.
- **ndims** (`int`) – The number of batch dimensions in the result.
- **event_inputs** (`tuple`) – A tuple of names to assign to rightmost dimensions.

Returns a multivariate normal distribution.

Return type `pyro.distributions.MultivariateNormal`

funsor_to_cat_and_mvn (*funsor_*, *ndims*, *event_inputs*)

Converts a labeled gaussian mixture model to a pair of distributions.

Parameters

- **funsor** (`funsor.joint.Joint`) – A Gaussian mixture funsor.
- **ndims** (`int`) – The number of batch dimensions in the result.

Returns A pair (`cat`, `mvn`), where `cat` is a `Categorical` distribution over mixture components and `mvn` is a `MultivariateNormal` with rightmost batch dimension ranging over mixture components.

class AffineNormal (*matrix*, *loc*, *scale*, *value_x*, *value_y*)

Bases: *funsor.terms.Funsor*

Represents a conditional diagonal normal distribution over a random variable Y whose mean is an affine function of a random variable X . The likelihood of X is thus:

```
AffineNormal(matrix, loc, scale).condition(y).log_density(x)
```

which is equivalent to:

```
Normal(x @ matrix + loc, scale).to_event(1).log_prob(y)
```

Parameters

- **matrix** (*Funsor*) – A transformation from X to Y . Should have rightmost shape (`x_dim`, `y_dim`).
- **loc** (*Funsor*) – A constant offset for Y 's mean. Should have output shape (`y_dim`,).
- **scale** (*Funsor*) – Standard deviation for Y . Should have output shape (`y_dim`,).

- **value_x**(Funsor) – A value X.
- **value_y**(Funsor) – A value Y.

matrix_and_mvn_to_funsor (*matrix*, *mvn*, *event_dims*=(), *x_name*='value_x', *y_name*='value_y')

Convert a noisy affine function to a Gaussian. The noisy affine function is defined as:

```
y = x @ matrix + mvn.sample()
```

The result is a non-normalized Gaussian funsor with two real inputs, *x_name* and *y_name*, corresponding to a conditional distribution of real vector *y* given real vector *x*.

Parameters

- **matrix** (*torch.Tensor*) – A matrix with rightmost shape (*x_size*, *y_size*).
- **mvn** (*torch.distributions.MultivariateNormal* or *torch.distributions.Independent of torch.distributions.Normal*) – A multivariate normal distribution with *event_shape* == (*y_size*,).
- **event_dims** (*tuple*) – A tuple of names for rightmost dimensions. These will be assigned to *result.inputs* of type *Bint*.
- **x_name** (*str*) – The name of the *x* random variable.
- **y_name** (*str*) – The name of the *y* random variable.

Returns A funsor with given *real_inputs* and possibly additional *Bint* inputs.

Return type *funsor.terms.Funsor*

Distribution Funsors

This interface provides a number of standard normalized probability distributions implemented as funsors.

class `Distribution(*args)`

Bases: `funsor.terms.Funsor`

Funsor backed by a PyTorch/JAX distribution object.

Parameters `*args` – Distribution-dependent parameters. These can be either funsors or objects that can be coerced to funsors via `to_funsor()`. See derived classes for details.

dist_class = 'defined by derived classes'

eager_reduce (`op, reduced_vars`)

has_enumerate_support

classmethod **eager_log_prob** (`*params`)

unscaled_sample (`sampled_vars, sample_inputs, rng_key=None`)

enumerate_support (`expand=False`)

entropy ()

mean ()

variance ()

class `Beta(concentration1, concentration0, value='value')`

Bases: `funsor.distribution.Distribution`

dist_class

alias of `pyro.distributions.torch.Beta`

class `Cauchy(loc, scale, value='value')`

Bases: `funsor.distribution.Distribution`

dist_class

alias of `pyro.distributions.torch.Cauchy`

```
class Chi2 (df, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Chi2

class BernoulliProbs (probs, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of funsor.torch.distributions._PyroWrapper_BernoulliProbs

class BernoulliLogits (logits, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of funsor.torch.distributions._PyroWrapper_BernoulliLogits

class Binomial (total_count, probs, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Binomial

class Categorical (probs, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Categorical

class CategoricalLogits (logits, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of funsor.torch.distributions._PyroWrapper_CategoricalLogits

class Delta (v, log_density, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.delta.Delta

class Dirichlet (concentration, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Dirichlet

class DirichletMultinomial (concentration, total_count, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.conjugate.DirichletMultinomial

class Exponential (rate, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Exponential

class Gamma (concentration, rate, value='value')
    Bases: funsor.distribution.Distribution
```

```

    dist_class
        alias of pyro.distributions.torch.Gamma

class GammaPoisson (concentration, rate, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.conjugate.GammaPoisson

class Geometric (probs, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Geometric

class Gumbel (loc, scale, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Gumbel

class HalfCauchy (scale, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.HalfCauchy

class HalfNormal (scale, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.HalfNormal

class Laplace (loc, scale, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Laplace

class LowRankMultivariateNormal (loc, cov_factor, cov_diag, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.LowRankMultivariateNormal

class Multinomial (total_count, probs, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Multinomial

class MultivariateNormal (loc, scale_tril, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.MultivariateNormal

class NonreparameterizedBeta (concentration1, concentration0, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.testing.fakes.NonreparameterizedBeta

```

```
class NonreparameterizedDirichlet (concentration, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.testing.fakes.NonreparameterizedDirichlet

class NonreparameterizedGamma (concentration, rate, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.testing.fakes.NonreparameterizedGamma

class NonreparameterizedNormal (loc, scale, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.testing.fakes.NonreparameterizedNormal

class Normal (loc, scale, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Normal

class Pareto (scale, alpha, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Pareto

class Poisson (rate, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Poisson

class StudentT (df, loc, scale, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.StudentT

class Uniform (low, high, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.Uniform

class VonMises (loc, concentration, value='value')
    Bases: funsor.distribution.Distribution

    dist_class
        alias of pyro.distributions.torch.VonMises
```

Mini-Pyro Interface

This interface provides a backend for the Pyro probabilistic programming language. This interface is intended to be used indirectly by writing standard Pyro code and setting `pyro_backend("functor")`. See `examples/minipyro.py` for example usage.

12.1 Mini Pyro

This file contains a minimal implementation of the Pyro Probabilistic Programming Language. The API (method signatures, etc.) match that of the full implementation as closely as possible. This file is independent of the rest of Pyro, with the exception of the `pyro.distributions` module.

An accompanying example that makes use of this implementation can be found at `examples/minipyro.py`.

```
class Distribution (functor_dist, sample_inputs=None)
```

```
    Bases: object
```

```
    log_prob (value)
```

```
    expand_inputs (name, size)
```

```
get_param_store ()
```

```
class Messenger (fn=None)
```

```
    Bases: object
```

```
    process_message (msg)
```

```
    postprocess_message (msg)
```

```
class trace (fn=None)
```

```
    Bases: functor.minipyro.Messenger
```

```
    postprocess_message (msg)
```

```
    get_trace (*args, **kwargs)
```

```
class replay (fn, guide_trace)
    Bases: funsor.minipyro.Messenger

    process_message (msg)

class block (fn=None, hide_fn=<function block.<lambda>>)
    Bases: funsor.minipyro.Messenger

    process_message (msg)

class seed (fn=None, rng_seed=None)
    Bases: funsor.minipyro.Messenger

class CondIndepStackFrame (name, size, dim)
    Bases: tuple

    dim
        Alias for field number 2

    name
        Alias for field number 0

    size
        Alias for field number 1

class PlateMessenger (fn, name, size, dim)
    Bases: funsor.minipyro.Messenger

    process_message (msg)

tensor_to_funsor (value, cond_indep_stack, output)

class log_joint (fn=None)
    Bases: funsor.minipyro.Messenger

    process_message (msg)

    postprocess_message (msg)

apply_stack (msg)

sample (name, fn, obs=None, infer=None)

param (name, init_value=None, constraint=Real(), event_dim=None)

plate (name, size, dim)

class PyroOptim (optim_args)
    Bases: object

class Adam (optim_args)
    Bases: funsor.minipyro.PyroOptim

    TorchOptimizer
        alias of torch.optim.adam.Adam

class ClippedAdam (optim_args)
    Bases: funsor.minipyro.PyroOptim

    TorchOptimizer
        alias of pyro.optim.clipped_adam.ClippedAdam

class SVI (model, guide, optim, loss)
    Bases: object

    step (*args, **kwargs)
```

Expectation (*log_probs, costs, sum_vars, prod_vars*)

elbo (*model, guide, *args, **kwargs*)

class ELBO (***kwargs*)

Bases: `object`

class Trace_ELBO (***kwargs*)

Bases: `funsor.minipyro.ELBO`

class TraceMeanField_ELBO (***kwargs*)

Bases: `funsor.minipyro.ELBO`

class TraceEnum_ELBO (***kwargs*)

Bases: `funsor.minipyro.ELBO`

class Jit (*fn, **kwargs*)

Bases: `object`

class Jit_ELBO (*elbo, **kwargs*)

Bases: `funsor.minipyro.ELBO`

JitTrace_ELBO (***kwargs*)

JitTraceMeanField_ELBO (***kwargs*)

JitTraceEnum_ELBO (***kwargs*)

Einsum Interface

This interface implements tensor variable elimination among tensors. In particular it does not implement continuous variable elimination.

naive_contract_einsum(*eqn*, **terms*, ***kwargs*)

Use for testing Contract against einsum

naive_einsum(*eqn*, **terms*, ***kwargs*)

Implements standard variable elimination.

naive_plated_einsum(*eqn*, **terms*, ***kwargs*)

Implements Tensor Variable Elimination (Algorithm 1 in [Obermeyer et al 2019])

[Obermeyer et al 2019] Obermeyer, F., Bingham, E., Jankowiak, M., Chiu, J., Pradhan, N., Rush, A., and Goodman, N. Tensor Variable Elimination for Plated Factor Graphs, 2019

einsum(*eqn*, **terms*, ***kwargs*)

Top-level interface for optimized tensor variable elimination.

Parameters

- **equation** (*str*) – An einsum equation.
- ***terms** (*functor.terms.Functor*) – One or more operands.
- **plates** (*set*) – Optional keyword argument denoting which functor dimensions are plate dimensions. Among all input dimensions (from terms): dimensions in plates but not in outputs are product-reduced; dimensions in neither plates nor outputs are sum-reduced.

CHAPTER 14

Indices and tables

- `genindex`
- `modindex`
- `search`

f

- `functor.adjoint`, 27
- `functor.affine`, 33
- `functor.cnf`, 23
- `functor.delta`, 18
- `functor.domains`, 9
- `functor.einsum`, 53
- `functor.gaussian`, 22
- `functor.integrate`, 24
- `functor.interpreter`, 11
- `functor.joint`, 23
- `functor.memoize`, 12
- `functor.minipyro`, 49
- `functor.montecarlo`, 12
- `functor.ops.array`, 5
- `functor.ops.builtin`, 2
- `functor.ops.op`, 1
- `functor.optimizer`, 25
- `functor.pyro.convert`, 42
- `functor.pyro.distribution`, 37
- `functor.pyro.hmm`, 38
- `functor.sum_product`, 29
- `functor.tensor`, 19
- `functor.terms`, 13
- `functor.testing`, 35
- `functor.torch.distributions`, 45

A

- `abs` (in module `functor.ops.builtin`), 2
- `abs()` (*Functor method*), 15
- `AbsOp` (class in `functor.ops.builtin`), 3
- `ActualExpected` (class in `functor.testing`), 35
- `Adam` (class in `functor.minipyro`), 50
- `add` (in module `functor.ops.builtin`), 2
- `AddOp` (class in `functor.ops.array`), 6
- `AddOp` (class in `functor.ops.builtin`), 3
- `adjoint()` (*AdjointTape method*), 27
- `adjoint_binary()` (in module `functor.adjoint`), 27
- `adjoint_cat()` (in module `functor.adjoint`), 27
- `adjoint_contract()` (in module `functor.adjoint`), 27
- `adjoint_contract_generic()` (in module `functor.adjoint`), 27
- `adjoint_contract_unary()` (in module `functor.adjoint`), 27
- `adjoint_reduce()` (in module `functor.adjoint`), 27
- `adjoint_subs_gaussian_gaussian()` (in module `functor.adjoint`), 27
- `adjoint_subs_gaussianmixture_discrete()` (in module `functor.adjoint`), 27
- `adjoint_subs_gaussianmixture_gaussianmixture()` (in module `functor.adjoint`), 27
- `adjoint_subs_tensor()` (in module `functor.adjoint`), 27
- `adjoint_tensor()` (in module `functor.adjoint`), 27
- `AdjointTape` (class in `functor.adjoint`), 27
- `affine_inputs()` (in module `functor.affine`), 33
- `AffineNormal` (class in `functor.pyro.convert`), 43
- `align()` (*Contraction method*), 23
- `align()` (*Delta method*), 19
- `align()` (*Functor method*), 14
- `align()` (*Gaussian method*), 23
- `align()` (*Tensor method*), 19
- `align_gaussian()` (in module `functor.gaussian`), 22
- `align_tensor()` (in module `functor.tensor`), 20
- `align_tensors()` (in module `functor.tensor`), 20
- `all` (in module `functor.ops.array`), 5
- `all()` (*Functor method*), 15
- `AllOp` (class in `functor.ops.array`), 6
- `amax` (in module `functor.ops.array`), 5
- `AmaxOp` (class in `functor.ops.array`), 6
- `amin` (in module `functor.ops.array`), 5
- `AminOp` (class in `functor.ops.array`), 6
- `and_` (in module `functor.ops.builtin`), 2
- `AndOp` (class in `functor.ops.builtin`), 3
- `any` (in module `functor.ops.array`), 5
- `any()` (*Functor method*), 15
- `AnyOp` (class in `functor.ops.array`), 6
- `apply_optimizer()` (in module `functor.optimizer`), 25
- `apply_stack()` (in module `functor.minipyro`), 50
- `arg_constraints` (*FunctorDistribution attribute*), 37
- `arg_constraints` (*GaussianHMM attribute*), 39
- `arg_constraints` (*SwitchingLinearHMM attribute*), 41
- `as_tensor()` (*BlockMatrix method*), 22
- `as_tensor()` (*BlockVector method*), 22
- `assert_close()` (in module `functor.testing`), 35
- `assert_equiv()` (in module `functor.testing`), 35
- `AssociativeOp` (class in `functor.ops.array`), 6
- `AssociativeOp` (class in `functor.ops.builtin`), 2
- `astype` (in module `functor.ops.array`), 5
- `AstypOp` (class in `functor.ops.array`), 6
- `atanh` (in module `functor.ops.builtin`), 2
- `atanh()` (*Functor method*), 15
- `AtanhOp` (class in `functor.ops.array`), 6
- `AtanhOp` (class in `functor.ops.builtin`), 3

B

- `BernoulliLogits` (class in `functor.torch.distributions`), 46
- `BernoulliProbs` (class in `functor.torch.distributions`), 46
- `Beta` (class in `functor.torch.distributions`), 45
- `Binary` (class in `functor.terms`), 16
- `binary_divide()` (in module `functor.cnf`), 24
- `binary_subtract()` (in module `functor.cnf`), 24

`binary_to_contract()` (in module `funsor.cnf`), 24
`Binomial` (class in `funsor.torch.distributions`), 46
`Bint` (class in `funsor.domains`), 9
`bint()` (in module `funsor.domains`), 10
`BintType` (class in `funsor.domains`), 9
`block` (class in `funsor.minipyro`), 50
`BlockMatrix` (class in `funsor.gaussian`), 22
`BlockVector` (class in `funsor.gaussian`), 22

C

`CachedOpMeta` (class in `funsor.ops.op`), 1
`Cat` (class in `funsor.terms`), 17
`cat` (in module `funsor.ops.array`), 5
`Categorical` (class in `funsor.torch.distributions`), 46
`CategoricalLogits` (class in `funsor.torch.distributions`), 46
`CatOp` (class in `funsor.ops.array`), 6
`Cauchy` (class in `funsor.torch.distributions`), 45
`check_funsor()` (in module `funsor.testing`), 35
`Chi2` (class in `funsor.torch.distributions`), 45
`cholesky` (in module `funsor.ops.array`), 5
`cholesky_inverse` (in module `funsor.ops.array`), 5
`cholesky_solve` (in module `funsor.ops.array`), 5
`clamp` (in module `funsor.ops.array`), 5
`clamp_finite()` (Tensor method), 19
`ClampOp` (class in `funsor.ops.array`), 6
`ClippedAdam` (class in `funsor.minipyro`), 50
`CondIndepStackFrame` (class in `funsor.minipyro`), 50
`Contraction` (class in `funsor.cnf`), 23

D

`declare_op_types()` (in module `funsor.ops.op`), 1
`Delta` (class in `funsor.delta`), 19
`Delta` (class in `funsor.torch.distributions`), 46
`detach` (in module `funsor.ops.array`), 5
`diagonal` (in module `funsor.ops.array`), 5
`DiagonalOp` (class in `funsor.ops.array`), 6
`dim` (`CondIndepStackFrame` attribute), 50
`Dirichlet` (class in `funsor.torch.distributions`), 46
`DirichletMultinomial` (class in `funsor.torch.distributions`), 46
`DiscreteHMM` (class in `funsor.pyro.hmm`), 38
`dispatch()` (`funsor.interpreter.StatefulInterpretation` class method), 12
`dispatch()` (`funsor.montecarlo.MonteCarlo` class method), 12
`dispatched_interpretation()` (in module `funsor.interpreter`), 11
`dist_class` (`BernoulliLogits` attribute), 46
`dist_class` (`BernoulliProbs` attribute), 46
`dist_class` (`Beta` attribute), 45
`dist_class` (`Binomial` attribute), 46
`dist_class` (`Categorical` attribute), 46

`dist_class` (`CategoricalLogits` attribute), 46
`dist_class` (`Cauchy` attribute), 45
`dist_class` (`Chi2` attribute), 46
`dist_class` (`Delta` attribute), 46
`dist_class` (`Dirichlet` attribute), 46
`dist_class` (`DirichletMultinomial` attribute), 46
`dist_class` (`Distribution` attribute), 45
`dist_class` (`Exponential` attribute), 46
`dist_class` (`Gamma` attribute), 46
`dist_class` (`GammaPoisson` attribute), 47
`dist_class` (`Geometric` attribute), 47
`dist_class` (`Gumbel` attribute), 47
`dist_class` (`HalfCauchy` attribute), 47
`dist_class` (`HalfNormal` attribute), 47
`dist_class` (`Laplace` attribute), 47
`dist_class` (`LowRankMultivariateNormal` attribute), 47
`dist_class` (`Multinomial` attribute), 47
`dist_class` (`MultivariateNormal` attribute), 47
`dist_class` (`NonreparameterizedBeta` attribute), 47
`dist_class` (`NonreparameterizedDirichlet` attribute), 48
`dist_class` (`NonreparameterizedGamma` attribute), 48
`dist_class` (`NonreparameterizedNormal` attribute), 48
`dist_class` (`Normal` attribute), 48
`dist_class` (`Pareto` attribute), 48
`dist_class` (`Poisson` attribute), 48
`dist_class` (`StudentT` attribute), 48
`dist_class` (`Uniform` attribute), 48
`dist_class` (`VonMises` attribute), 48
`dist_to_funsor()` (in module `funsor.pyro.convert`), 42
`distribute_subs_contraction()` (in module `funsor.cnf`), 24
`Distribution` (class in `funsor.distribution`), 45
`Distribution` (class in `funsor.minipyro`), 49
`do_fresh_subs()` (in module `funsor.cnf`), 24
`Domain` (in module `funsor.domains`), 9
`dtype` (`Bint` attribute), 9
`dtype` (`Funsor` attribute), 14
`dtype` (`RealsType` attribute), 9

E

`eager()` (in module `funsor.terms`), 13
`eager_contract_base()` (in module `funsor.optimizer`), 25
`eager_contraction_gaussian()` (in module `funsor.cnf`), 24
`eager_contraction_generic_recursive()` (in module `funsor.cnf`), 23
`eager_contraction_generic_to_tuple()` (in module `funsor.cnf`), 23

[eager_contraction_tensor\(\)](#) (in module [fun-
sor.cnf](#)), 24
[eager_contraction_to_binary\(\)](#) (in module
[funsor.cnf](#)), 23
[eager_contraction_to_reduce\(\)](#) (in module
[funsor.cnf](#)), 23
[eager_independent_joint\(\)](#) (in module [fun-
sor.joint](#)), 23
[eager_log_prob\(\)](#) ([funsor.distribution.Distribution](#)
class method), 45
[eager_markov_product\(\)](#) (in module [fun-
sor.sum_product](#)), 31
[eager_or_die\(\)](#) (in module [funsor.terms](#)), 13
[eager_reduce\(\)](#) ([Delta](#) method), 19
[eager_reduce\(\)](#) ([Distribution](#) method), 45
[eager_reduce\(\)](#) ([Funsor](#) method), 15
[eager_reduce\(\)](#) ([Gaussian](#) method), 23
[eager_reduce\(\)](#) ([Stack](#) method), 17
[eager_reduce\(\)](#) ([Tensor](#) method), 19
[eager_reduce_exp\(\)](#) (in module [funsor.joint](#)), 23
[eager_subs\(\)](#) ([Cat](#) method), 17
[eager_subs\(\)](#) ([Delta](#) method), 19
[eager_subs\(\)](#) ([Funsor](#) method), 15
[eager_subs\(\)](#) ([Gaussian](#) method), 23
[eager_subs\(\)](#) ([Independent](#) method), 18
[eager_subs\(\)](#) ([MarkovProduct](#) method), 31
[eager_subs\(\)](#) ([Slice](#) method), 17
[eager_subs\(\)](#) ([Stack](#) method), 17
[eager_subs\(\)](#) ([Tensor](#) method), 19
[eager_subs\(\)](#) ([Variable](#) method), 16
[eager_unary\(\)](#) ([Funsor](#) method), 15
[eager_unary\(\)](#) ([Number](#) method), 17
[eager_unary\(\)](#) ([Tensor](#) method), 19
[Einsum](#) (class in [funsor.tensor](#)), 21
[einsum](#) (in module [funsor.ops.array](#)), 5
[einsum\(\)](#) (in module [funsor.einsum](#)), 53
[EinsumOp](#) (class in [funsor.ops.array](#)), 6
[ELBO](#) (class in [funsor.minipyro](#)), 51
[elbo\(\)](#) (in module [funsor.minipyro](#)), 51
[empty\(\)](#) (in module [funsor.testing](#)), 35
[entropy\(\)](#) ([Distribution](#) method), 45
[entropy\(\)](#) ([Independent](#) method), 18
[enumerate_support\(\)](#) ([Distribution](#) method), 45
[eq](#) (in module [funsor.ops.builtin](#)), 2
[EqOp](#) (class in [funsor.ops.builtin](#)), 3
[exp](#) (in module [funsor.ops.builtin](#)), 2
[exp\(\)](#) ([Funsor](#) method), 15
[expand](#) (in module [funsor.ops.array](#)), 5
[expand\(\)](#) ([DiscreteHMM](#) method), 38
[expand\(\)](#) ([FunsorDistribution](#) method), 37
[expand\(\)](#) ([SwitchingLinearHMM](#) method), 41
[expand_inputs\(\)](#) ([Distribution](#) method), 49
[Expectation\(\)](#) (in module [funsor.minipyro](#)), 50
[Exponential](#) (class in [funsor.torch.distributions](#)), 46

[ExpOp](#) (class in [funsor.ops.array](#)), 6
[ExpOp](#) (class in [funsor.ops.builtin](#)), 3
[extract_affine\(\)](#) (in module [funsor.affine](#)), 33

F

[filter\(\)](#) ([SwitchingLinearHMM](#) method), 41
[find_domain\(\)](#) (in module [funsor.domains](#)), 10
[finfo](#) (in module [funsor.ops.array](#)), 5
[full_like](#) (in module [funsor.ops.array](#)), 5
[Full_likeOp](#) (class in [funsor.ops.array](#)), 6
[Function](#) (class in [funsor.tensor](#)), 20
[function\(\)](#) (in module [funsor.tensor](#)), 21
[Funsor](#) (class in [funsor.terms](#)), 13
[funsor.adjoint](#) (module), 27
[funsor.affine](#) (module), 33
[funsor.cnf](#) (module), 23
[funsor.delta](#) (module), 18
[funsor.domains](#) (module), 9
[funsor.einsum](#) (module), 53
[funsor.gaussian](#) (module), 22
[funsor.integrate](#) (module), 24
[funsor.interpreter](#) (module), 11
[funsor.joint](#) (module), 23
[funsor.memoize](#) (module), 12
[funsor.minipyro](#) (module), 49
[funsor.montecarlo](#) (module), 12
[funsor.ops.array](#) (module), 5
[funsor.ops.builtin](#) (module), 2
[funsor.ops.op](#) (module), 1
[funsor.optimizer](#) (module), 25
[funsor.pyro.convert](#) (module), 42
[funsor.pyro.distribution](#) (module), 37
[funsor.pyro.hmm](#) (module), 38
[funsor.sum_product](#) (module), 29
[funsor.tensor](#) (module), 19
[funsor.terms](#) (module), 13
[funsor.testing](#) (module), 35
[funsor.torch.distributions](#) (module), 45
[funsor_to_cat_and_mvn\(\)](#) (in module [fun-
sor.pyro.convert](#)), 43
[funsor_to_mvn\(\)](#) (in module [funsor.pyro.convert](#)),
43
[funsor_to_tensor\(\)](#) (in module [fun-
sor.pyro.convert](#)), 42
[FunsorDistribution](#) (class in [fun-
sor.pyro.distribution](#)), 37
[funsordistribution_to_funsor\(\)](#) (in module
[funsor.pyro.distribution](#)), 37

G

[Gamma](#) (class in [funsor.torch.distributions](#)), 46
[GammaPoisson](#) (class in [funsor.torch.distributions](#)), 47
[Gaussian](#) (class in [funsor.gaussian](#)), 22
[GaussianHMM](#) (class in [funsor.pyro.hmm](#)), 38

GaussianMixture (in module *funsor.cnf*), 23
 GaussianMRF (class in *funsor.pyro.hmm*), 39
 ge (in module *funsor.ops.builtin*), 2
 Geometric (class in *funsor.torch.distributions*), 47
 GeOp (class in *funsor.ops.builtin*), 3
 get_param_store() (in module *funsor.minipyro*), 49
 get_trace() (trace method), 49
 getitem (in module *funsor.ops.builtin*), 2
 GetitemOp (class in *funsor.ops.builtin*), 3
 gt (in module *funsor.ops.builtin*), 2
 GtOp (class in *funsor.ops.builtin*), 3
 Gumbel (class in *funsor.torch.distributions*), 47

H

HalfCauchy (class in *funsor.torch.distributions*), 47
 HalfNormal (class in *funsor.torch.distributions*), 47
 has_enumerate_support (Distribution attribute), 45
 has_rsample (DiscreteHMM attribute), 38
 has_rsample (GaussianHMM attribute), 39
 has_rsample (GaussianMRF attribute), 40
 has_rsample (SwitchingLinearHMM attribute), 41

I

id_from_inputs() (in module *funsor.testing*), 35
 ignore_jit_warnings() (in module *funsor.tensor*), 19
 Independent (class in *funsor.terms*), 18
 input_vars (Funsor attribute), 14
 Integrate (class in *funsor.integrate*), 24
 interpretation() (in module *funsor.interpreter*), 11
 inv() (TransformOp static method), 1, 4
 invert (in module *funsor.ops.builtin*), 2
 InvertOp (class in *funsor.ops.builtin*), 3
 is_affine() (in module *funsor.affine*), 33
 is_array() (in module *funsor.testing*), 35
 is_numeric_array (in module *funsor.ops.array*), 5
 isnan (in module *funsor.ops.array*), 5
 IsnanOp (class in *funsor.ops.array*), 6
 item() (Funsor method), 14
 item() (Number method), 17
 item() (Tensor method), 19

J

Jit (class in *funsor.minipyro*), 51
 Jit_ELBO (class in *funsor.minipyro*), 51
 JitTrace_ELBO() (in module *funsor.minipyro*), 51
 JitTraceEnum_ELBO() (in module *funsor.minipyro*), 51
 JitTraceMeanField_ELBO() (in module *funsor.minipyro*), 51

L

Lambda (class in *funsor.terms*), 17
 Laplace (class in *funsor.torch.distributions*), 47
 lazy() (in module *funsor.terms*), 13
 le (in module *funsor.ops.builtin*), 2
 LeOp (class in *funsor.ops.builtin*), 3
 lgamma (in module *funsor.ops.builtin*), 2
 LgammaOp (class in *funsor.ops.builtin*), 3
 log (in module *funsor.ops.builtin*), 2
 log() (Funsor method), 15
 loglp (in module *funsor.ops.builtin*), 2
 loglp() (Funsor method), 15
 LoglpOp (class in *funsor.ops.array*), 6
 LoglpOp (class in *funsor.ops.builtin*), 3
 log_abs_det_jacobian() (TransformOp static method), 1, 4
 log_joint (class in *funsor.minipyro*), 50
 log_normalizer (Gaussian attribute), 23
 log_prob() (DiscreteHMM method), 38
 log_prob() (Distribution method), 49
 log_prob() (FunsorDistribution method), 37
 log_prob() (SwitchingLinearHMM method), 41
 logaddexp (in module *funsor.ops.array*), 5
 LogaddexpOp (class in *funsor.ops.array*), 6
 LogOp (class in *funsor.ops.array*), 6
 LogOp (class in *funsor.ops.builtin*), 3
 logsumexp (in module *funsor.ops.array*), 5
 logsumexp() (Funsor method), 15
 LowRankMultivariateNormal (class in *funsor.torch.distributions*), 47
 lt (in module *funsor.ops.builtin*), 2
 LtOp (class in *funsor.ops.builtin*), 3

M

make_chain_einsum() (in module *funsor.testing*), 36
 make_einsum_example() (in module *funsor.testing*), 35
 make_hmm_einsum() (in module *funsor.testing*), 36
 make_op() (in module *funsor.ops.op*), 1
 make_plated_hmm_einsum() (in module *funsor.testing*), 36
 MarkovProduct (class in *funsor.sum_product*), 31
 MarkovProductMeta (class in *funsor.sum_product*), 31
 materialize() (Tensor method), 20
 matmul (in module *funsor.ops.builtin*), 2
 MatmulOp (class in *funsor.ops.builtin*), 3
 matrix_and_mvn_to_funsor() (in module *funsor.pyro.convert*), 44
 max (in module *funsor.ops.builtin*), 2
 max() (Funsor method), 15
 MaxOp (class in *funsor.ops.array*), 6
 MaxOp (class in *funsor.ops.builtin*), 3

[mean\(\)](#) (*Distribution method*), 45
[mean\(\)](#) (*Independent method*), 18
[memoize\(\)](#) (*in module funsor.memoize*), 12
[Messenger](#) (*class in funsor.minipyro*), 49
[min](#) (*in module funsor.ops.builtin*), 2
[min\(\)](#) (*Funsor method*), 15
[MinOp](#) (*class in funsor.ops.array*), 6
[MinOp](#) (*class in funsor.ops.builtin*), 3
[mixed_sequential_sum_product\(\)](#) (*in module funsor.sum_product*), 30
[modified_partial_sum_product\(\)](#) (*in module funsor.sum_product*), 29
[moment_matching\(\)](#) (*in module funsor.terms*), 13
[moment_matching_contract_default\(\)](#) (*in module funsor.joint*), 23
[moment_matching_contract_joint\(\)](#) (*in module funsor.joint*), 23
[moment_matching_reduce\(\)](#) (*Funsor method*), 15
[MonteCarlo](#) (*class in funsor.montecarlo*), 12
[mul](#) (*in module funsor.ops.builtin*), 2
[MulOp](#) (*class in funsor.ops.builtin*), 3
[Multinomial](#) (*class in funsor.torch.distributions*), 47
[MultivariateNormal](#) (*class in funsor.torch.distributions*), 47
[mvn_to_funsor\(\)](#) (*in module funsor.pyro.convert*), 42

N

[naive_contract_einsum\(\)](#) (*in module funsor.einsum*), 53
[naive_einsum\(\)](#) (*in module funsor.einsum*), 53
[naive_plated_einsum\(\)](#) (*in module funsor.einsum*), 53
[naive_sarkka_bilmes_product\(\)](#) (*in module funsor.sum_product*), 31
[naive_sequential_sum_product\(\)](#) (*in module funsor.sum_product*), 30
[name](#) (*CondIndepStackFrame attribute*), 50
[ne](#) (*in module funsor.ops.builtin*), 2
[neg](#) (*in module funsor.ops.builtin*), 2
[NegOp](#) (*class in funsor.ops.builtin*), 4
[NeOp](#) (*class in funsor.ops.builtin*), 4
[new_arange](#) (*in module funsor.ops.array*), 5
[new_arange\(\)](#) (*Tensor method*), 19
[new_eye](#) (*in module funsor.ops.array*), 5
[new_zeros](#) (*in module funsor.ops.array*), 5
[NonreparameterizedBeta](#) (*class in funsor.torch.distributions*), 47
[NonreparameterizedDirichlet](#) (*class in funsor.torch.distributions*), 47
[NonreparameterizedGamma](#) (*class in funsor.torch.distributions*), 48
[NonreparameterizedNormal](#) (*class in funsor.torch.distributions*), 48

[Normal](#) (*class in funsor.torch.distributions*), 48
[normalize_contraction_commutative_canonical_order\(\)](#) (*in module funsor.cnf*), 24
[normalize_contraction_commute_joint\(\)](#) (*in module funsor.cnf*), 24
[normalize_contraction_generic_args\(\)](#) (*in module funsor.cnf*), 24
[normalize_contraction_generic_tuple\(\)](#) (*in module funsor.cnf*), 24
[normalize_fuse_subs\(\)](#) (*in module funsor.cnf*), 24
[normalize_trivial\(\)](#) (*in module funsor.cnf*), 24
[NullOp](#) (*class in funsor.ops.builtin*), 3
[nullop](#) (*in module funsor.ops.builtin*), 2
[Number](#) (*class in funsor.terms*), 17

O

[of_shape\(\)](#) (*in module funsor.terms*), 18
[ones\(\)](#) (*in module funsor.testing*), 35
[Op](#) (*class in funsor.ops.array*), 6
[Op](#) (*class in funsor.ops.builtin*), 4
[Op](#) (*class in funsor.ops.op*), 1
[optimize\(\)](#) (*in module funsor.optimizer*), 25
[optimize_contract_finitary_funsor\(\)](#) (*in module funsor.optimizer*), 25
[optimize_contraction_variadic\(\)](#) (*in module funsor.optimizer*), 25
[or_](#) (*in module funsor.ops.builtin*), 2
[OrOp](#) (*class in funsor.ops.builtin*), 4

P

[param\(\)](#) (*in module funsor.minipyro*), 50
[Pareto](#) (*class in funsor.torch.distributions*), 48
[partial_sum_product\(\)](#) (*in module funsor.sum_product*), 29
[partial_unroll\(\)](#) (*in module funsor.sum_product*), 29
[PatternMissingError](#), 12
[permute](#) (*in module funsor.ops.array*), 5
[plate\(\)](#) (*in module funsor.minipyro*), 50
[PlateMessenger](#) (*class in funsor.minipyro*), 50
[Poisson](#) (*class in funsor.torch.distributions*), 48
[postprocess_message\(\)](#) (*log_joint method*), 50
[postprocess_message\(\)](#) (*Messenger method*), 49
[postprocess_message\(\)](#) (*trace method*), 49
[pow](#) (*in module funsor.ops.builtin*), 2
[PowOp](#) (*class in funsor.ops.builtin*), 4
[pretty\(\)](#) (*Funsor method*), 14
[process_message\(\)](#) (*block method*), 50
[process_message\(\)](#) (*log_joint method*), 50
[process_message\(\)](#) (*Messenger method*), 49
[process_message\(\)](#) (*PlateMessenger method*), 50
[process_message\(\)](#) (*replay method*), 50
[prod](#) (*in module funsor.ops.array*), 5

`prod()` (*Funsor method*), 15
`ProdOp` (*class in funsor.ops.array*), 7
`PyroOptim` (*class in funsor.minipyro*), 50

Q

`quote()` (*Funsor method*), 14

R

`rand()` (*in module funsor.testing*), 35
`randint()` (*in module funsor.testing*), 35
`randn()` (*in module funsor.testing*), 35
`random_gaussian()` (*in module funsor.testing*), 35
`random_mvn()` (*in module funsor.testing*), 36
`random_scale_tril()` (*in module funsor.testing*), 35
`random_tensor()` (*in module funsor.testing*), 35
`Real` (*class in funsor.domains*), 9
`Reals` (*class in funsor.domains*), 9
`reals()` (*in module funsor.domains*), 10
`RealsType` (*class in funsor.domains*), 9
`reciprocal` (*in module funsor.ops.builtin*), 2
`ReciprocalOp` (*class in funsor.ops.array*), 7
`ReciprocalOp` (*class in funsor.ops.builtin*), 4
`recursion_reinterpret_contraction()` (*in module funsor.cnf*), 23
`Reduce` (*class in funsor.terms*), 16
`reduce()` (*Funsor method*), 14
`reduce_funsor()` (*in module funsor.cnf*), 24
`reflect()` (*in module funsor.terms*), 13
`register()` (*funsor.interpreter.StatefulInterpretation class method*), 12
`registry` (*MonteCarlo attribute*), 12
`registry` (*StatefulInterpretation attribute*), 12
`reinterpret()` (*in module funsor.interpreter*), 11
`replay` (*class in funsor.minipyro*), 49
`requires_grad` (*Funsor attribute*), 14
`requires_grad` (*Tensor attribute*), 19
`reshape()` (*Funsor method*), 15
`ReshapeOp` (*class in funsor.ops.array*), 6
`rsample()` (*FunsorDistribution method*), 37

S

`safediv` (*in module funsor.ops.builtin*), 2
`SafedivOp` (*class in funsor.ops.array*), 7
`SafedivOp` (*class in funsor.ops.builtin*), 4
`safesub` (*in module funsor.ops.builtin*), 2
`SafesubOp` (*class in funsor.ops.array*), 7
`SafesubOp` (*class in funsor.ops.builtin*), 4
`sample` (*in module funsor.ops.array*), 5
`sample()` (*Funsor method*), 14
`sample()` (*FunsorDistribution method*), 37
`sample()` (*in module funsor.minipyro*), 50
`SampleOp` (*class in funsor.ops.array*), 7

`sarkka_bilmes_product()` (*in module funsor.sum_product*), 31
`seed` (*class in funsor.minipyro*), 50
`sequential()` (*in module funsor.terms*), 13
`sequential_reduce()` (*Funsor method*), 15
`sequential_sum_product()` (*in module funsor.sum_product*), 30
`set_interpretation()` (*in module funsor.interpreter*), 11
`set_inv()` (*TransformOp method*), 1, 4
`set_log_abs_det_jacobian()` (*TransformOp method*), 1, 4
`shape` (*Bint attribute*), 9
`shape` (*Funsor attribute*), 14
`shape` (*Real attribute*), 9
`shape` (*Reals attribute*), 9
`sigmoid` (*in module funsor.ops.builtin*), 2
`sigmoid()` (*Funsor method*), 15
`SigmoidOp` (*class in funsor.ops.builtin*), 4
`size` (*BintType attribute*), 9
`size` (*CondIndepStackFrame attribute*), 50
`Slice` (*class in funsor.terms*), 17
`SoftplusOp` (*class in funsor.ops.builtin*), 4
`solve()` (*in module funsor.delta*), 18
`sqrt` (*in module funsor.ops.builtin*), 2
`sqrt()` (*Funsor method*), 15
`SqrtOp` (*class in funsor.ops.array*), 7
`SqrtOp` (*class in funsor.ops.builtin*), 4
`Stack` (*class in funsor.terms*), 17
`stack` (*in module funsor.ops.array*), 5
`stack()` (*in module funsor.tensor*), 22
`StackOp` (*class in funsor.ops.array*), 7
`StatefulInterpretation` (*class in funsor.interpreter*), 11
`step()` (*SVI method*), 50
`StudentT` (*class in funsor.torch.distributions*), 48
`sub` (*in module funsor.ops.builtin*), 2
`subclass_register()` (*funsor.ops.array.Op class method*), 7
`subclass_register()` (*funsor.ops.builtin.Op class method*), 4
`subclass_register()` (*funsor.ops.op.Op class method*), 1
`SubOp` (*class in funsor.ops.builtin*), 4
`Subs` (*class in funsor.terms*), 16
`sum` (*in module funsor.ops.array*), 5
`sum()` (*Funsor method*), 15
`sum_product()` (*in module funsor.sum_product*), 30
`SumOp` (*class in funsor.ops.array*), 7
`support` (*FunsorDistribution attribute*), 37
`SVI` (*class in funsor.minipyro*), 50
`SwitchingLinearHMM` (*class in funsor.pyro.hmm*), 40

T

[tanh \(in module funsor.ops.builtin\)](#), 2
[tanh \(\) \(Funsor method\)](#), 15
[TanhOp \(class in funsor.ops.array\)](#), 7
[TanhOp \(class in funsor.ops.builtin\)](#), 4
[Tensor \(class in funsor.tensor\)](#), 19
[tensor_to_funsor \(\) \(in module funsor.minipyro\)](#), 50
[tensor_to_funsor \(\) \(in module funsor.pyro.convert\)](#), 42
[tensordot \(\) \(in module funsor.tensor\)](#), 21
[to_data \(\) \(in module funsor.terms\)](#), 15
[to_funsor \(\) \(in module funsor.terms\)](#), 15
[TorchOptimizer \(Adam attribute\)](#), 50
[TorchOptimizer \(ClippedAdam attribute\)](#), 50
[trace \(class in funsor.minipyro\)](#), 49
[Trace_ELBO \(class in funsor.minipyro\)](#), 51
[TraceEnum_ELBO \(class in funsor.minipyro\)](#), 51
[TraceMeanField_ELBO \(class in funsor.minipyro\)](#), 51
[TransformOp \(class in funsor.ops.builtin\)](#), 4
[TransformOp \(class in funsor.ops.op\)](#), 1
[transpose \(in module funsor.ops.array\)](#), 5
[TransposeOp \(class in funsor.ops.array\)](#), 7
[triangular_solve \(in module funsor.ops.array\)](#), 5
[truediv \(in module funsor.ops.builtin\)](#), 2
[TruedivOp \(class in funsor.ops.builtin\)](#), 4

U

[Unary \(class in funsor.terms\)](#), 16
[unary_contract \(\) \(in module funsor.cnf\)](#), 24
[unary_log_exp \(\) \(in module funsor.cnf\)](#), 24
[unary_neg_variable \(\) \(in module funsor.cnf\)](#), 24
[UnaryOp \(class in funsor.ops.builtin\)](#), 4
[UnaryOp \(class in funsor.ops.op\)](#), 1
[unfold \(\) \(in module funsor.optimizer\)](#), 25
[unfold_contraction_generic_tuple \(\) \(in module funsor.optimizer\)](#), 25
[unfold_contraction_variadic \(\) \(in module funsor.optimizer\)](#), 25
[Uniform \(class in funsor.torch.distributions\)](#), 48
[unscaled_sample \(\) \(Contraction method\)](#), 23
[unscaled_sample \(\) \(Delta method\)](#), 19
[unscaled_sample \(\) \(Distribution method\)](#), 45
[unscaled_sample \(\) \(Funsor method\)](#), 14
[unscaled_sample \(\) \(Gaussian method\)](#), 23
[unscaled_sample \(\) \(Independent method\)](#), 18
[unscaled_sample \(\) \(Subs method\)](#), 16
[unscaled_sample \(\) \(Tensor method\)](#), 19
[unsqueeze \(in module funsor.ops.array\)](#), 5

V

[Variable \(class in funsor.terms\)](#), 16

[variance \(\) \(Distribution method\)](#), 45
[variance \(\) \(Independent method\)](#), 18
[VonMises \(class in funsor.torch.distributions\)](#), 48

X

[xfail_if_not_found \(\) \(in module funsor.testing\)](#), 35
[xfail_if_not_implemented \(\) \(in module funsor.testing\)](#), 35
[xfail_param \(\) \(in module funsor.testing\)](#), 35
[xor \(in module funsor.ops.builtin\)](#), 2
[XorOp \(class in funsor.ops.builtin\)](#), 5

Z

[zeros \(\) \(in module funsor.testing\)](#), 35